Volatility capital buffer to prevent the breach of the Solvency II capital requirements*

Zoltán Zubor

The Solvency II regulation prescribes continuous capital adequacy, despite the fact that insurance companies only determine their capital adequacy in a reliable manner once annually. The volatility capital buffer[1] (VCB) is meant to guarantee that, despite the higher volatility arising from the market valuation, at a given α confidence level the solvency capital of insurers meets the capital requirement on a continuous basis. This paper reduces the problem to the search of the probability distribution quantile belonging to the α confidence level (VaR_α), the 99.5 per cent quantile of which is the solvency capital requirement (SCR) specified in the Solvency II Capital Regulation, and thus the VCB can be expressed as a percentage of the SCR. Without the assumptions related to the distribution, any value may be obtained for the VCB ratio, but it can be squeezed into a relatively narrow band even under natural assumptions. On the one hand, the analysis of these distribution groups may further narrow the possible values, and on the other hand it points out that in the case of fatter-tailed distributions (when major, extreme losses may also occur more frequently) and positive skewness (when the probability of the loss is smaller than that of the profit, but the value thereof is expected to be higher), we obtain a lower VCB ratio.

Journal of Economic Literature (JEL) Classification: C13, C46, G22

Keywords: Solvency II, volatility, capital buffer, loss ddistribution, quantile, confidence level

1. Introduction

The two most important quantitative elements of the Solvency II regime, which entered into force on 1 January 2016, is the changeover to market valuation and the capital requirements covering all risks of insurers. Insurance liabilities have no market and hence they also have no market price. The new regime models the value at which another insurer would accept the liabilities. In the case of the solvency capital requirement, the value to be defined is the one under which the

* The views expressed in this paper are those of the author(s) and do not necessarily reflect the offical view of the Magyar Nemzeti Bank.

Zoltán Zubor is the chief actuary of the Magyar Nemzeti Bank. E-mail: zuborz@mnb.hu.

[1] The notion of volatility capital buffer appeared in the communication of the Magyar Nemzeti Bank for the first time in 2014.

maximum probability of the decrease in the insurer's solvency capital to a greater degree than this is 0.5 per cent.

The changeover to market valuation implies greater short-term volatility of the solvency capital and capital adequacy (*EIOPA 2011; EIOPA 2013*). The short-term high volatility of capital adequacy, and through that of the insurers' financial position is not in line with the typically long-term nature of the business (*Insurance Europe 2013*). There were several ideas for the elimination of artificial volatility, which were tested by the European Insurance and Occupational Pensions Authority (EIOPA) in the LTGA[2] impact study in 2013 (*EIOPA 2013*). In the present regulation, the smoothing of artificial volatility is served by the volatility adjustment, the matching adjustment and – in the initial period – the transitional measures (LTG measures).

According to the EIOPA 2014 stress test, the impact of the LTG measures is ambiguous. Although the individual elements may exert a significant impact on the capital adequacy,[3] only a few insurers made or could make use of the opportunity: the volatility adjustment, the matching adjustment and the various transitional measures were applied by 31, 7 and 2–10 per cent of the participants, respectively (*EIOPA 2014*).

The higher volatility of the Solvency II capital adequacy also impacts the Hungarian market (*MNB 2015a*), which was confirmed by the impact studies as well (MNB 2015b, Bora et al. 2015). Based on the data of 11 insurers[4] that participated in each of the last five impact studies,[5] the average of the relative standard deviation[6] of their Solvency I capital adequacy ratios is 0.179, while it is 0.260 in the case of the Solvency II ratios, which clearly reflects that capital adequacy in the new regime is substantially more volatile.

The LTG measures have a modest impact in the Hungarian market. According to the quantitative impact studies performed by the Magyar Nemzeti Bank in 2014, none of the participating insurers[7] have applied matching adjustment or wish to apply it in the future. All of them presented the impact of the volatility adjustment, but this resulted in a mere 4.1 per cent improvement in the capital level, i.e. the LTG measures are able to absorb the high artificial volatility of the Solvency II capital

[2] Long-Term Guarantee Assessment

[3] For example, the capital level of those that applied the volatility adjustment increased on average by almost 30 per cent.

[4] Capital-proportional coverage: 64–75%.

[5] QIS5 (*EIOPA 2009*); QIS5bis (*HFSA 2010*); QIS_2012 (*HFSA 2012*); QIS_2014 (*MNB 2014*); RIGL (*data supply for preparation purposes 2015*) – in each case on the data of the end of the year preceding the year of the implementation of the impact analysis.

[6] The quotient of the variance and the expected value.

[7] 23 insurers – 80% coverage in proportion to capital.

adequacy only to a small degree, and do not eliminate the risks arising from the high volatility.

Article 100 of Directive 138/2009/EC (hereinafter: Directive) and (in accordance with this) Section 99 of Act 88/2014 (hereinafter: Insurance Act) prescribe continuous capital adequacy as a general rule, while insurers are only obliged to determine and report their compliance with the prescribed capital requirement periodically: the solvency capital requirement is to be presented annually, while the solvency capital is to be reported quarterly. How can this compliance be guaranteed in the interim periods? The regulation offers an ambiguous solution for this. The insurer need only comply with the last reported solvency capital requirement, which must be recalculated during the year as well, if the risk profile of the insurer changes materially. Although the trends in solvency capital must be monitored continuously and reported quarterly, the review based on exhaustive, audited data is usually performed only annually; i.e. due to the high volatility it may easily occur that an insurer with adequate, e.g. 120 per cent, capital level becomes short of capital even within a year, thereby violating the law.

If the insurer or the supervisory authority wishes to reduce the risk of capital shortfall in the interim period that lasts until the next reliable calculation of the insurer's capital position, it is practicable to hold slightly higher capital than the capital requirement (capital buffer) or to prescribe this for the insurers.

So far we have no example of management of the risks arising from the higher volatility using a capital buffer (volatility capital buffer), and thus there is also no literature on this. The topic and notion of the volatility capital buffer was first raised in November 2014 by Koppány Nagy (MNB) at the MABISZ (Association of Hungarian Insurance Companies) conference. One of the most important objectives of this paper is to determine and introduce the purpose[8] and exact content of the volatility capital buffer (VCB).

The second most important purpose of the paper is to present an approach that permits the reduction of the VCB (through the probability distribution underlying the SCR, as discussed later) to the SCR value. It is shown that in the absence of an assumption with regard to said distribution any value may be obtained for the VCB, but subject to various natural assumptions the possible buffer rates can be reasonably limited. The analysis of different distribution families may serve as a basis to determine the capital buffer to be maintained or which is worth maintaining, when the risk profile of the insurers corresponds to the given distribution family

[8] The volatility of the capital adequacy may be attributable to several factors the volatility capital buffer does not have to respond to all factors, as in certain cases – e.g. when the portfolio changes substantially or contrary to the preliminary expectations, or the insurer changes its internal procedures or calculation models in a way that may significantly impact the capital adequacy – the insurer can be expected to perform an extraordinary determination and presentation of its capital position.

the most. On the other hand, it highlights the fact that we obtain a smaller capital buffer in the case of those insurers where there is a higher probability of extreme losses,[9] or where the value of the expected loss is likely to exceed the value of the expected profit, i.e. the assumption of normal distribution usually results in an upper estimate for the VCB value.

2. Purpose of the volatility capital buffer

The purpose of the volatility capital buffer is to reduce the risk of the insurer experiencing a capital shortfall in the interim period between the reliable calculations and presentations of the capital adequacy.

The capital adequacy of an insurer may change due to several factors. These factors can be grouped based on three criteria:

i. By portfolio: Whether the change occurred due to the change in the existing portfolio, in the new portfolio that developed in accordance with the preliminary expectations or due to change in the portfolio that significantly departs from the expectations. The existing and new portfolio means the portfolio that already existed on the reference date of the last reliable capital adequacy report or acquired thereafter – until the next calculation – also bearing in mind the contract boundaries.

ii. By core components: Whether the improvement/deterioration occurred due to the change in the capital requirement or in the eligible solvency capital.

iii. By type of trigger: Whether the change in the capital level occurred due to external or internal causes. Internal factors include all events that are attributable to the insurer or the owners, such as changing the models, assumptions or processes used for the determination of certain balance sheet items, capital elements or the SCR, payment of dividends or capital replenishment.

According to Section 268(1) of the Insurance Act and Section 27 of Government Decree 43/2015 (hereinafter: Decree 43), the insurer must perform the extraordinary calculation of its capital requirement if its risk profile has changed significantly, e.g. if its portfolio has changed in a significantly different manner and degree than expected. In this case (also bearing in mind Section 27(5) of Decree 43[10]), the insurer is expected to recalculate its entire capital adequacy and report it to the supervisory authority. The insurer may also be expected to recalculate and report its capital adequacy, if it introduces such new models or assumptions that substantially influence the capital adequacy.

[9] So-called fat-tailed distributions, or right-skewed distributions.

[10] Based on which the supervisory authority may oblige the insurer to recalculate its capital requirement or (to accumulate solvency capital of sufficient volume), if there is a good reason to assume that the risk profile of the insurer has changed.

Based on the foregoing, the volatility capital buffer must respond only to the existing and the expected new portfolio, and it needs to absorb only the risks arising from those changes in the solvency capital that occurred due to a change in the external factor (environmental changes).

The solvency capital is the sum of the basic and the ancillary own funds. Basic own funds comprise the assets exceeding the liabilities (net asset value) and the subordinated liabilities. In Hungary, the role of the ancillary capital is marginal. On the other hand, it is distinct and not exposed to random volatility. The latter statement is also valid for the subordinated liabilities, i.e. of the solvency capital only the change in the net asset value has relevance for the VCB.

In avoiding the capital shortfall, it is not recommended – and usually also not possible – to aim for 100 per cent certainty. The volatility capital buffer means the surplus capital that over the given horizon (according to the above: one year) provides protection against the volatility of the basic own funds at the α confidence level ($0\% < \alpha < 100\%$) and ensures permanent capital adequacy in accordance with the laws. To be more precise, VCB_α is the value, where *(1)*

$$P\left(X < VCB_\alpha\right) = \alpha, \tag{1}$$

where the *X* probability variable is the *decrease* in the value of the basic own funds within a given time horizon, due to external factors, in respect of the existing and expected to be acquired new insurance portfolio.[11] VCB_α is the quantile of the *X* probability variable belonging to level α, or – with the term also used in the Insurance Act – its value-at-risk *(2)*

$$VCB_\alpha = VaR_\alpha\left(X\right). \tag{2}$$

According to Article 101(3) of the Directive, "The Solvency Capital Requirement shall be calibrated so as to ensure that all quantifiable risks to which an insurance or reinsurance company is exposed are taken into account. *It shall cover existing business, as well as the new business expected to be written over the following 12 months. With respect to existing business, it shall cover only unexpected losses.* It shall correspond to the value-at-risk of the core solvency capital of an insurance or reinsurance company subject to a confidence level of 99.5 % over a one-year period." According to this formulation the *X* probability variable included in the definition of the VCB of a one-year horizon corresponds to the probability variable that is also included in the definition of the solvency capital requirement, the value belonging to the 99.5 per cent quantile (VaR) of which is the SCR, i.e. *(3)*

$$VCB_{99.5\%} = SRC. \tag{3}$$

[11] Equation (1) has a solution for all α, if the *X* probability variable is absolutely continuous. In our case this may be assumed.

With this, we reduced the task to the search for the quantile belonging to a given confidence level of such a probability variable, the 99.5 per cent quantile of which is known to us (at least in theory; see below). In order to see clearly what this approach really means and the type of risks managed by the VCB thus obtained, it must be clarified that the SCR (in theory) is the 99.5 per cent value-at-risk of what kind of *X* probability variable.

According to the Solvency II regulation, *X* means the loss incurred on the existing portfolio and on the portfolio expected to be acquired in the next 12 months. By calculating technical provisions, all possible future cash flows (with their own probability) arising from the existing portfolio (within the contract boundaries) must be taken into consideration. Based on this, *X* means the *unexpected loss* in respect of the existing portfolio. This is also confirmed by Article 101(3) of the Directive. However, no such condition is included in the laws in respect of the portfolio to be acquired in the next 12 month, and thus in this respect the expected loss (which in fact is a profit in the case of most insurers) must be also taken into consideration. However, there is no trace of this in the standard formula; therefore in the following let us assume that the solvency capital requirement provides cover for the *unexpected losses* in respect of the new portfolio as well, i.e. the expected value of the loss (of the *X* probability variable) underlying the definition of the SCR (with the standard formula) is zero.

The question is whether we want to consider the expected profitability of the new portfolio, and if so, how to do so. Based on the outlined objectives, the volatility capital buffer needs to respond only to the unexpected part; however, upon its application it must be borne in mind that the expected profit/loss is not at all accidental. For example, a home insurance contract should be typically treated as one that will be terminated on the next renewal date. However, the majority of the contracts are automatically renewed (which, according to the contract boundaries, qualifies as new contract), thus in the case of a profitable portfolio the new portfolio to be acquired in the next 12 months presumably will be also profitable.

Hereafter, the unexpected loss serves as the basis for the volatility capital buffer, i.e. $VCB_\alpha = VaR_\alpha\left(X\right)$, where *X* is the *unexpected decrease* in basic own funds due to changes in the environment, in respect of the existing portfolio and the new insurance portfolio to be acquired in the next 12 months, similarly to the conceptual definition of the standard formula of SCR.

Accordingly, we look for the α quantile of such a probability variable (VCB_α), the 99.5 per cent quantile of which is known to us (SCR). However, does the SCR defined with the standard formula indeed correspond to the 99.5 per cent quantile of the given insurer's actual *X* probability variable? For this each of the following conditions should be satisfied: *(i)* the standard formula is well-calibrated, *(ii)* the standard formula describes the risk profile of the given insurance undertaking accurately, *(iii)* the insurer calculated its capital requirement precisely, in accordance with the standard formula, based on real, reliable data.

Of these, the first two conditions are definitely not satisfied: let us merely consider the correlation coefficients defined as the multiples of 0.25, or the flood risk factors defined identically within a county. It can be, and is perhaps worth disputing how well the standard formula measures the risks, but not in connection with the volatility capital buffer; hence, in the following I assume that the SCR calculated and reported by the insurer is the 99.5 per cent quantile of exactly that probability variable the α quantile of which we are looking for.

The Solvency II regime prescribes a two-tier capital requirement. The breach of the minimum capital requirement (MCR) – which is usually of lower degree,[12] can easily be calculated and must be determined quarterly – entails substantially stricter supervisory measures. The more stringent (higher) solvency capital requirement (SCR) must be defined by a complex model, which wishes to respond to all possible risks, annually. It is worth mapping the volatility capital buffer with this two-tier system, with different confidence levels: a higher level should be targeted in the case of the MCR.

3. Possible approaches for the calculation of the volatility capital buffer

3.1. Based on the distribution of the total unexpected loss

As outlined in Section 1 $VCB_\alpha = VaR_\alpha(X)$, where X – the unexpected decrease in basic own funds – is the same probability variance, the 99.5 per cent quantile of which is the SCR.

If we knew the distribution of X, it would be easy to define the VCB_α. However, the unexpected loss occurs as a result of various shocks, under dependency relations, and thus it is not possible to determine the distribution accurately. Moreover, in the case of insurers with contracts of different claim distribution, different reinsurance coverage, different asset portfolio, etc., the attributes of the unexpected loss distribution may fundamentally differ from each other.

Approximation of the capital buffer may be performed by using different assumptions for the type of distribution (distribution family), estimating the necessary parameters. Here we may rely on the fact that the SCR is the value-at-risk of the same distribution belonging to the 99.5 per cent level, on a one-year horizon.

[12] The MCR can be defined by a relatively simple formula, but it must not exceed 45 per cent of the SCR (i.e. it is lower than the SCR), but an absolute threshold depending on the activity must be reached, e.g. in the case of life insurers EUR 3.7 million (i.e. in the case of smaller insurers this lower threshold may be higher than the SCR).

Setting out from the $VCB_{99,5\%}=SCR$ fundamental assumption, lower and upper estimates for the VCB may be performed based on more general assumptions for the distribution, which may help assess the result obtained with assumptions for the various distribution families.

3.2. Based on the modular decomposition of the unexpected loss

It can be seen that the value of the capital buffer to be obtained on the basis of the total unexpected loss depends significantly on the distribution of the loss. Due to the different business models of the individual insurers, this distribution may be distinctly different, and this difference may also substantially influence the estimation of the VCB; there are also such artificial business models in the case of which the different distributions generate extremely different capital buffers (see Section 4.2.1).

We may try to remedy this problem – with substantial extra work – by decomposing the loss function into components corresponding to the modules of the SCR standard formula, defining the appropriate volatility capital buffer part for the individual components and aggregating them by applying the appropriate correlations.

This method will return a more accurate and reliable result only if we know the distribution of the unexpected loss belonging to the individual modules. But let us just look at the non-life insurance catastrophe risk module as a basis: the loss distribution essentially varies depending whether it has a proportional or non-proportional reinsurance coverage.

Thus, the problem discussed in the first half of this section – which we tried to solve with the modular approach – may also occur in the case of the individual modules. And, although it is possible that for certain modules there is a better foundation for assuming the distribution of the unexpected loss, and thereby the VCB can be estimated with a lower error margin in the case of the individual modules, the aggregation of the sub-results is problematic. Although there are given correlations necessary for the aggregation in the SCR standard formula, those belong to the 99.5 per cent confidence level (99.5 per cent VaR), but nothing guarantees that the same correlations are suitable in the case of a confidence level of 75 per cent, for example. The different diversification impact may significantly distort the final result.

Let us take, for example, the *X* and *Y* marginal distributions of the bivariate *(X; Y)* uniform distribution on [-0.5;0.5]*X*[-0.5;0.5]. These are independent probability variables, of uniform distribution on the *[*-0.5;0.5*]* interval, with expected value of zero, the quartile of which belonging to *α* equals to *α-0.5*. It is easy to see that the appropriate quantile of the *Z=X+Y* probability variable is $1-\sqrt{2\cdot(1-\alpha)}$. If we want to aggregate $\sqrt{VaR_X^2+2\cdot\rho\cdot VaR_X\cdot VaR_Y+VaR_Y^2}$ then in the case of *α=99,5%* the aggregation should be performed by *ρ=0,653*, while in the case of *α=75%* it should be performed by *ρ=0,314*. An aggregation by 0.653 would distort the result upward by 55.2 per cent.

On the other hand, it is more important to set lower and upper bounds for the targeted capital level than to make a more accurate estimate, i.e. to be able to say whether the given estimate is a lower or upper estimate. It is still a question whether we do not lose more with the modular approach on the problem of aggregation than we can gain by the more accurate estimation of the individual modules. At present this is an open question.

In summary: The modular approach does not decrease the reliability of the estimated volatility capital buffer significantly, while the multiple distorting effects lower the transparency of the potential disharmony between the hypothetical value and the estimate; as such I do not go into further details on this approach.

3.3. Empirical approach

The appropriate quantile of the loss function may also be estimated with the use of empirical data related to the change in the solvency capital or the capital position (capital surplus).

The usability of the empirical VaR requires relatively many observed values, i.e. in our case we need long time series. For example, to ensure that the empirical quantile value belonging to the 90 per cent level is not determined automatically by the highest value, we would need at least 15–20 data points, meaning a time series of 15–20 years. This method should be excluded not only because we do not have such a long Solvency II time series, but also because the condition of usability is that the observed values should originate from probability variables of identical distribution, i.e. the risk profile of the insurer and the environment[13] should not change. This cannot be assumed even for a short time horizon.

Another possibility is to assume that the distribution of the unexpected loss belongs to a certain distribution family, and we estimate the necessary parameters of the assumed distribution from the available data. For example, the standard deviation of the distribution with the use of the empirical standard deviation. The estimation of the missing parameters returns a reliable result only if we have a sufficiently large number of estimations, i.e. sufficiently long time series. However, the invariance of the distribution cannot be guaranteed on the longer horizon, while this is also a condition for the applicability of the method.

Another condition in both cases is that the observed values should be independent of each other. It is questionable whether the annual values of the unexpected losses may be deemed independent.

The capital buffer linked to empirical standard deviation is a logical choice, as it is the standard deviation of the unexpected losses that best characterises the volatility

[13] The unexpected loss depends not only on the insurer's portfolio and operation, but also on the environment.

against which the VCB protects the insurer from the capital shortfall. However, without knowing the distribution (distribution type) of the loss we cannot judge the level (probability) of the protection provided by e.g. a capital buffer of 2/3 (empirical) variance. In the case of normal distribution this protection is 75 per cent, but under another distribution this may be overly or insufficiently prudent. The aforementioned factors (sufficiently long time series, steadiness of the distribution and the environment, independence) question the adequacy of the estimate thus obtained even more.

In Section 4.3 I present empirical data despite the fact that, according to the foregoing, it is not possible to determine an adequate VCB based on those.

3.4. Time horizon

The purpose of the volatility capital buffer is to prevent capital shortfall in those interim periods when the insurer does not calculate its capital position. The minimum capital requirement and the solvency capital are to be determined quarterly, while the solvency capital requirement must be calculated annually. The insurers need to comply only with the last reported capital requirements, and thus VCB should not provide protection against the possible change in the capital requirements. This means that the "interim period" is the period when we have no information on the solvency capital, i.e. the capital buffer must provide sufficient protection on a quarterly time horizon. On the other hand, the insurers often perform accurate calculations for determining the value of some assets and the majority of the liabilities only annually, and they must have their data audited also annually only, which questions the reliability of the quarterly figures. The objectives of the capital buffer may also include the elimination of the uncertainties arising from the superficial estimate, which raises the necessity of the one-year horizon.

If X_i denotes the unexpected loss incurring in quarter i ($X = X_1 + X_2 + X_3 + X_4$), and we know the value of $VaR_\alpha(X)$, then under certain circumstances we may also determine the value of $VaR_\alpha(X_1)$. For example, if the VaR is proportionate to the variance (e.g. in the case of normal distribution) and we assume that the X_i are independent and of identical distribution, then *(4)*

$$VaR_\alpha\left(X_1\right)=\frac{VaR_\alpha\left(X\right)}{2}. \tag{4}$$

However, usually none of the proportionality, the independence and the identical distribution conditions is satisfied.

Let us assume that an insurer is sensitive only to the decline of the yield curve. The substantial, unexpected loss incurred in the first quarter means the yield curve significantly declined. However, in this case in the second quarter the yield curve can no longer decline to such an extent that make it suffer a loss of the same degree. Thus the value of X_1 also influences the distribution of X_2 and not only its value.

Hereafter we search for the VCB belonging to the one-year horizon.

4. Estimating the volatility capital buffer

The task is to determine the α quantile of a probability variable of zero expected value in the knowledge of its 99.5 per cent quantile. It can be seen that if we assume nothing on the distribution of X, then we can obtain any value for the $vcb_\alpha = \frac{VCB_\alpha}{SRC}$ proportion.

It is reasonable to set out from the assumption that X is of normal distribution. In this case, independently of the parameters of the specific distribution, we simply obtain the VCB values as a function of α. This may be considered as an initial, benchmark value; however, the distribution of an insurer's loss may significantly depart from the normal one. This is typically the case when the insurer underwrites significant risks or undertakes long-term obligations, which may give rise to material losses, while the magnitude of the profit is relatively limited (i.e. the distribution is a positively skewed[14]), or when the probability of the substantial losses is not negligible ("fat-tailed" [15] distribution). Hence it makes sense to examine other possible distribution families as well.

The real loss distributions belong to a given distribution family at least by better or worse approximation, thus the analysis of the constraints based on the more general features of the distribution may also be useful in terms of the practical application.

4.1. Based on assumptions related to the type of the loss distribution (distribution family)

This Section has a dual purpose. On one hand, it is possible to look at the VCB values that would be obtained if the type of the X probability variable was known. On the other hand, the results demonstrate that the right-skewed and fatter-tailed attributes decrease the rate of the capital buffer, i.e. the value obtained by assuming normality may be regarded as a kind of upper estimate.

The Section discusses the named distribution families in slightly more detail than absolutely necessary, to enable even those to interpret the obtained values (i.e.

[14] The skewness of a distribution may be determined in several ways. The most commonly accepted measure is the Pearson's skewness, which is nothing else but the third moment of the standardised distribution (expected value of its third power). The distribution is right-skewed, if its third central moment is positive. In the case of a loss distribution this means that the probability of an unexpected large loss is higher than that of an unexpected high profit.

[15] Intuitively, the distribution of Y loss is more fat-tailed than the X distribution, if in the case of Y the probability of extremely high losses is higher than in the case of X, and this relation increases with the increasingly higher losses.

in which case they may approximate the loss distributions of certain insurers, to what extent the result may be realistic) who are less familiar with the features of the individual distribution families.

4.1.1. Assuming that X is of normal distribution

In this case *(5)*

$$VCB_\alpha = \phi^{-1}(\alpha)\cdot\sigma + m, \tag{5}$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution function, σ is the standard deviation of X, m is the expected value of X . Based on $E(X) = 0$ $m = 0$.

On the other hand, (using $m = 0$)

$$SRC = \phi^{-1}(99.5\%)\cdot\sigma, \text{ from where} \tag{6}$$

$$\sigma = \frac{SCR}{\phi^{-1}(99.5\%)}. \text{ Substituting:} \tag{7}$$

$$VCB_\alpha = \frac{\phi^{-1}(\alpha)}{\phi^{-1}(99.5\%)}\cdot SCR \tag{8}$$

which practically means a capital adequacy requirement of $(1+vcb_\alpha)$ times, where

$$vcb_\alpha = \frac{\phi^{-1}(\alpha)}{\phi^{-1}(99.5\%)}. \tag{9}$$

The value of the capital buffer thus obtained can be easily determined; the values belonging to the individual levels are in presented in *Table 1.* For example, in the case of a capital adequacy of 126.2 per cent, the probability of the insurer's compliance with the (old) capital requirement even after one year is 75 per cent, while for achieving a confidence level of 90 per cent a capital level of almost 150 per cent is required.

Table 1.
Volatility capital buffer as a percentage of SCR – normal distribution

α	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
vcb_α	0.0%	4.9%	9.8%	15.0%	20.4%	26.2%	32.7%	40.2%	49.8%	63.9%

Source: Own calculations.

4.1.2. Assuming that X belongs to other named distribution families

Before discussing the named distribution families, let us make a few digressions. In respect of the *X* probability variable in question, we assume that its expected value is zero. However, at the majority of the possible distributions (e.g. exponential, lognormal, Pareto) it is $E(X)>0$. In this case either the quantiles of the $X'=X-E(X)$ transformed probability variable should be examined, or (which in fact is the same) the distance of the quantiles from the expected value.

In the remaining part of the Section we search for the (10) vcb_α capital buffer ratio:

$$vcb_\alpha = \frac{VCB_\alpha}{SCR} = \frac{VaR_\alpha(X)-E(X)}{VaR_{99.5\%}(X)-E(X)}. \tag{10}$$

If the *Y* probability variable is one time the constant of *X* (i.e. an insurer's unexpected losses always just coincide with e.g. seven times of another insurer's unexpected losses), then we get the same vcb_α value for *Y*, as $VaR_\alpha(c \cdot X)=c \cdot VaR_\alpha(X)$, and $E(c \cdot X)=c \cdot E(X)$, i.e. in (10) it may be reduced by c[16]. Consequently, the value of vcb_α is invariant to the linear transformation of the distribution (displacement invariance). (For example, it would have been enough in the previous Section as well to examine only the standard normal distribution.)

4.1.2.1. Skew normal distribution

The impact of the skewness on the capital buffer is illustrated through the skew normal distribution. Probability density function *(11)*

$$f(x)=2\varphi(x)\phi(ax)\ ^{17} \tag{11}$$

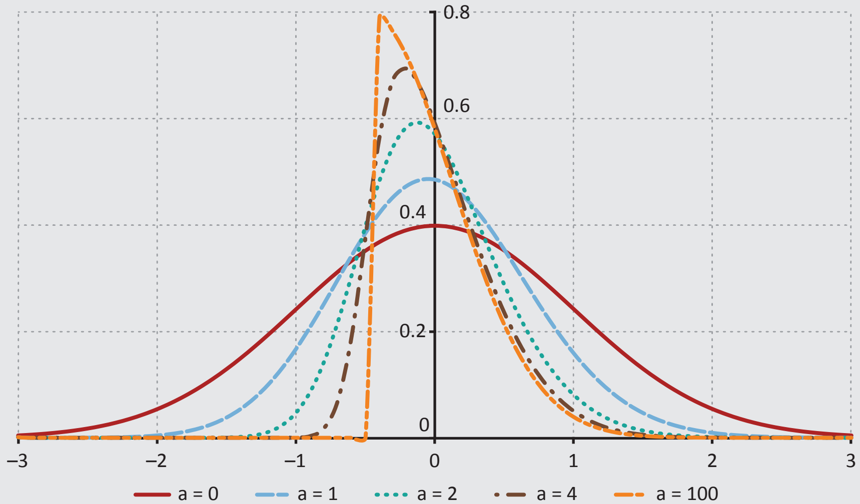
where φ and ϕ are the cumulative distribution function and probability density function of the standard normal distribution (for more details, see Azzalini1 – azzalini.stat.unipd.it). Parameter *a* determines the skewness of the distribution. $a = 0$ returns a standard normal distribution, $a > 0$ returns a positively skewed, while $a < 0$ returns a negatively skewed distribution. The greater the absolute value of the *a* parameter is, the more skewed the distribution will be.[18]

[16] Obviously here, the only possibility is $c>0$, as in the case of c<0 the loss turns into profit and vice versa.

[17] *Az Y = aX+b lineáris transzformált eloszlásfüggvénye* $g(x)=2\varphi\left(\frac{x-b}{a}\right)\phi\left(\alpha\left(\frac{x-b}{a}\right)\right)$. *Ezek alkotják a teljes ferde normális eloszláscsaládot. A volatilitási tőkepuffer mértéke azonban invariáns a lineáris transzformációra, ezért elég a standardizált verziót vizsgálni.*

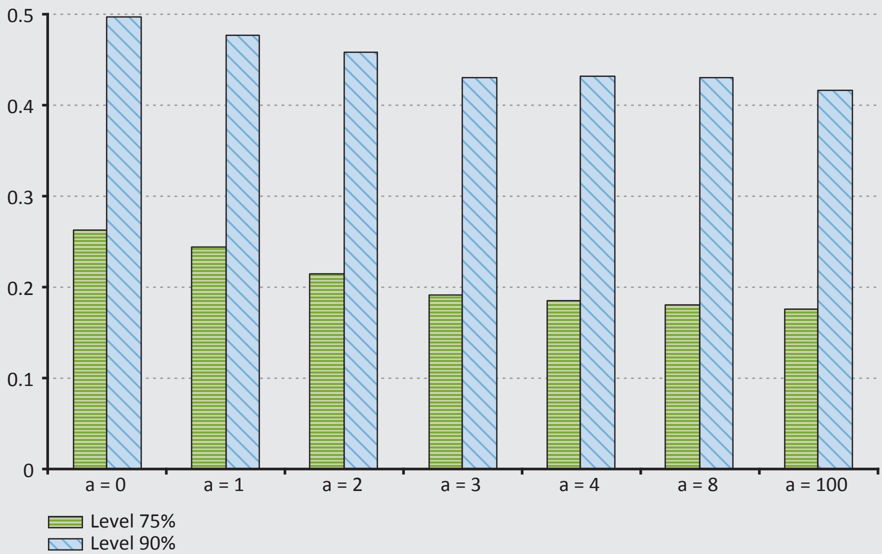
[18] Not only "visually", but also in mathematical terms.

Figure 1.
Probability density function of the skew normal distribution transformed to expected value of 0 and standard deviation of 1



Source: Own calculations.

Figure 2.
Value of the $vcb_{75\%}$ and $vcb_{90\%}$ under different a parameters



Source: Own calculations.

The simulations ran on one hundred thousand samples (based on Azzalini2 – azzalini.stat.unipd.it) clearly show (see Figure 2.) that the more positively skewed the distribution is, the smaller the vcb_α is. For example under *a = 4* even a 118.6 per cent capital adequacy provides a protection of 75 per cent, which in the case of normal distribution (*a = 0*) may only be achieved under a capital level of 126 per cent.

Table 2.
vcb_α values in the case of skewed normal distribution under different a parameters

	a = 0	a = 1	a = 2	a = 3	a = 4	a = 8	a = 100
65%	15.0%	13.6%	10.6%	8.6%	7.8%	7.2%	6.7%
75%	26.2%	24.5%	21.4%	19.0%	18.6%	18.0%	17.5%
85%	40.4%	38.0%	35.7%	33.2%	32.9%	32.7%	31.8%
95%	63.9%	61.9%	60.7%	58.3%	58.8%	58.6%	57.5%

Note: based on simulation run on one hundred thousand samples
Source: Own calculations.

According to *Arató (1995)*, the most often used claim distributions are the exponential, the lognormal, the Pareto, the gamma and the Weibull distributions; therefore it is worth examining the value of the volatility capital buffer with these distribution families as well, despite the fact that the volatility of claims is usually not the primary cause of the volatility of the net asset value.

4.1.2.2. Exponential distribution

Exponential distribution may be used for modelling the service life of equipment where the probability of breakdown does not depend on the age of the equipment ("ageless" distribution). Probability density function *(12)*

$$f(x)=\lambda e^{-\lambda x}\left(x>0\right) \tag{12}$$

wears off relatively fast, but it is significantly skewed to the right. Its expected value is *E(X)=1/λ*, thus the unexpected loss is *X-1/λ*. It can be easily deduced *(13)*

$$vcb_\alpha=\frac{\ln(1-\alpha)-1}{\ln(1-99.5\%)-1}, \tag{13}$$

i.e. vcb_α does not depend on the λ parameter. We knew this on the basis of the displacement invariance as well, since the changing of the λ parameter merely results in the linear transformation of the distribution.

Table 3. Volatility capital buffer as a percentage of SCR										
α	50%	55%	60%	65%	70%	75%	80%	85%	90%	95%
vcb_α	–7.1%	–4.7%	–1.9%	1.2%	4.7%	9.0%	14.2%	20.9%	30.3%	46.4%

Note: The negative vcb_α values are attributable to the fact, that (due to the strong skewness of the exponential distribution) VaR_α is lower than the expected value even under a relatively high confidence level.
Source: Own calculations.

4.1.2.3. Lognormal distribution

The distribution of a probability variable is lognormal when its logarithm is of normal distribution. Or in other words: if the *X* probability variable is of normal distribution, the e^X is of lognormal distribution. Accordingly, its probability density function *(14)*

$$f(x)=\frac{1}{\sigma\sqrt{2\pi x}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}\ (x>0). \tag{14}$$

This results in *(15)*

$$VaR_\alpha=e^{\phi^{-1}(\alpha)\sigma+\mu} \text{ and } E(X)=e^{\frac{\sigma^2}{2}+\mu}, \tag{15}$$

based on which *(16)*

$$vcb_\alpha=\frac{e^{\left(\phi^{-1}(\alpha)\sigma+\mu\right)}-e^{\frac{\sigma^2}{2}+\mu}}{e^{\left(\phi^{-1}(99.5\%)\sigma+\mu\right)}-e^{\frac{\sigma^2}{2}+\mu}}=\frac{e^{\phi^{-1}(\alpha)\sigma}-e^{\frac{\sigma^2}{2}}}{e^{\phi^{-1}(99.5\%)\sigma}-e^{\frac{\sigma^2}{2}}}, \tag{16}$$

i.e. vcb_α does not depend on μ (Φ is the cumulative distribution function of the standard normal distribution). This also follows from the displacement invariance, as the probability density function of $Y=\frac{X}{e^\mu}$ (17)

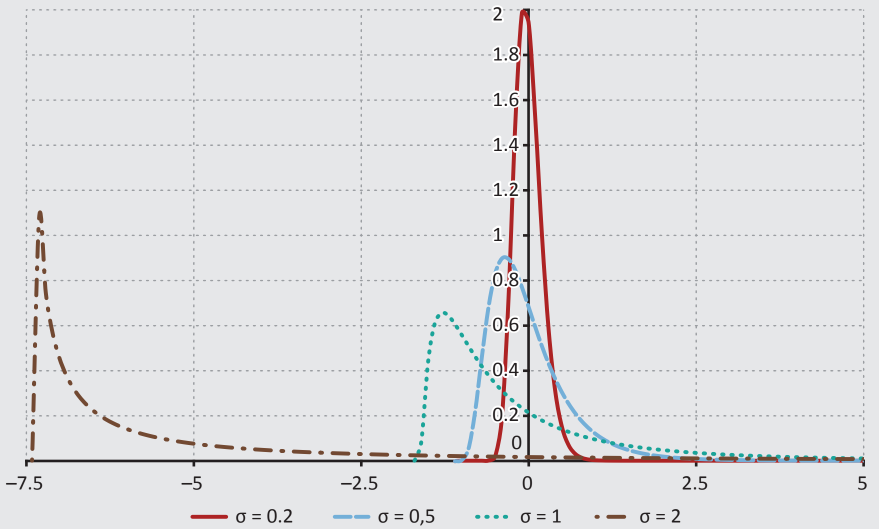
$$g(x)=\frac{1}{\sigma\sqrt{2\pi x}}e^{-\frac{(\ln x)^2}{2\sigma^2}}, \tag{17}$$

i.e. the $\mu = 0$ value can be obtained by linear transformation.

By increasing σ the distribution is increasingly skewed to the right. Pearson's skewness $\gamma=\sqrt{e^{\sigma^2}-1}\left(2+e^{\sigma^2}\right)$ increases extremely fast as a function of σ. In case of $\sigma<2\Phi^{-1}(\alpha)$ [19], based on the experiences (see Figure 4), *the value of vcb_α keeps getting smaller by increasing σ thus the skewness*.

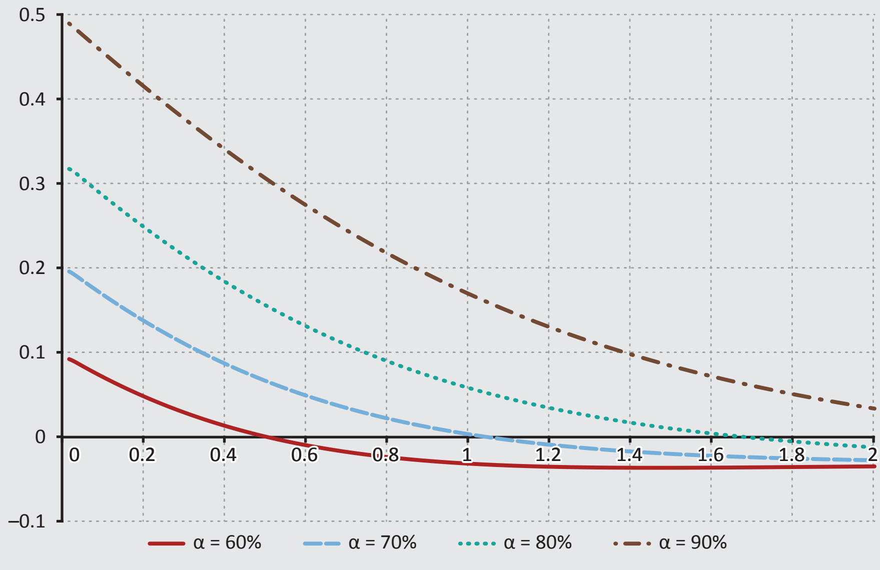
[19] In the case of $\sigma = 2$ the probability density function is already skewed to such a degree that even the 80 per cent quantile is smaller than the expected value, due to which $vcb_\alpha < 0$. We get a negative VCB, if $2\Phi^{-1}(99,5\%)>\sigma>2\Phi^{-1}(\alpha)$.

Figure 3.
Probability density function of the lognormal distribution shifted to zero expected value, under different σ parameters



Source: Own calculations.

Figure 4.
Value of vcb_{α} as a function of parameter σ



Source: Own calculations.

If we approximate to zero with σ, the skewness of the distribution is converging to zero, and the vcb_α values keep approximating (from below) the values obtained in the case of normal distribution, which is easy to prove formally as well, using (16) (which is also easy to deduce)

$$\lim_{\sigma\to 0} vcb_\alpha = \lim_{\sigma\to 0} \frac{e^{\phi^{-1}(\alpha)\cdot\sigma} - e^{\frac{\sigma^2}{2}}}{e^{\phi^{-1}(99.5\%)\cdot\sigma} - e^{\frac{\sigma^2}{2}}} = \frac{\phi^{-1}(\alpha)}{\phi^{1}(99.5\%)}, \tag{18}$$

which equals to the vcb_α obtained for the normal distribution, based on *(9).*

Table 4.
***vcb_α* values in the case of lognormal distribution under different σ parameters**

	σ = 1E-10	σ = 0,1	σ = 0,2	σ = 0,5	σ = 1	σ = 2
65%	15.0%	11.9%	9.2%	3.2%	–1.6%	–3.2%
75%	26.2%	22.4%	19.0%	10.8%	2.7%	–2.1%
85%	40.2%	36.1%	32.1%	21.9%	10.2%	0.3%
95%	63.9%	60.2%	56.5%	45.9%	30.7%	11.8%

Source: Own calculations.

4.1.2.4. Pareto distribution

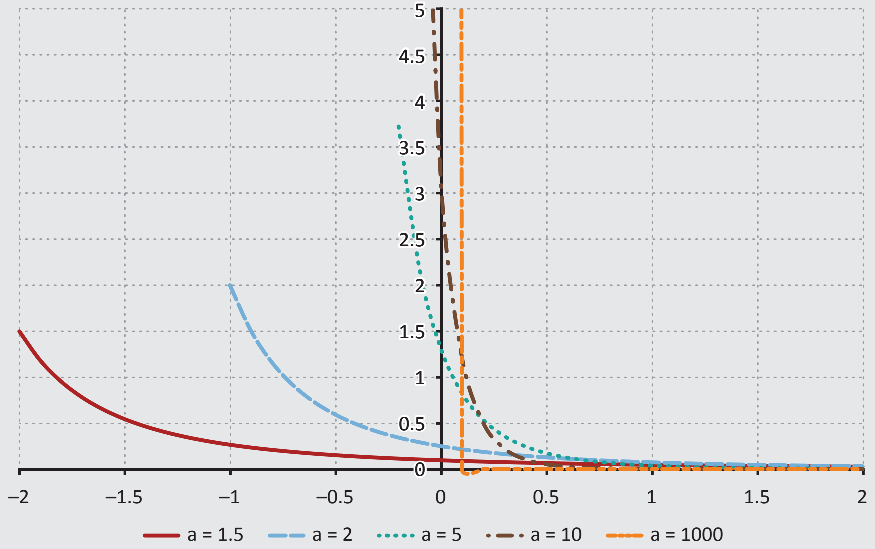
The Pareto distribution has a similar relation to the exponential distribution, as the lognormal to the normal one: if *X* is of *(a; c)* parameter Pareto distribution, then $\ln\left(\frac{X}{c}\right)$ is of *a* parameter exponential distribution (*Arató 1995*). Probability density function

$$f(x) = \frac{a\cdot c^a}{x^{a+1}}, \tag{19}$$

if $x > c$, otherwise 0. Changing parameter *c* simply means a linear transformation, which has no effect on the value of the capital buffer. It can be easily deduced *(20)*

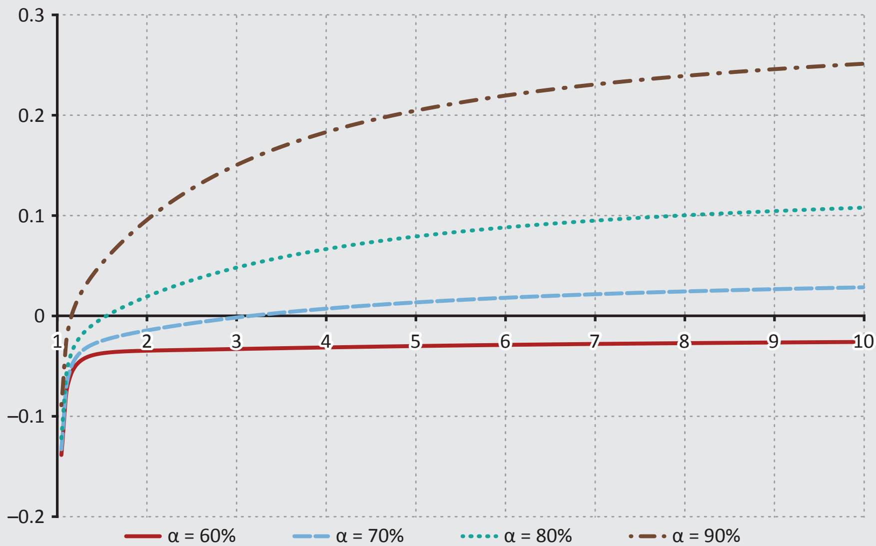
$$vcb_\alpha = \frac{(1-\alpha)^{-\frac{1}{a}} - \frac{a}{a-1}}{(1-0.995)^{-\frac{1}{a}} - \frac{a}{a-1}}. \tag{20}$$

Figure 5.
Probability density functions of the Pareto distribution shifted to zero expected value, under different *a*'s *(c = 1)*



Source: Own calculations.

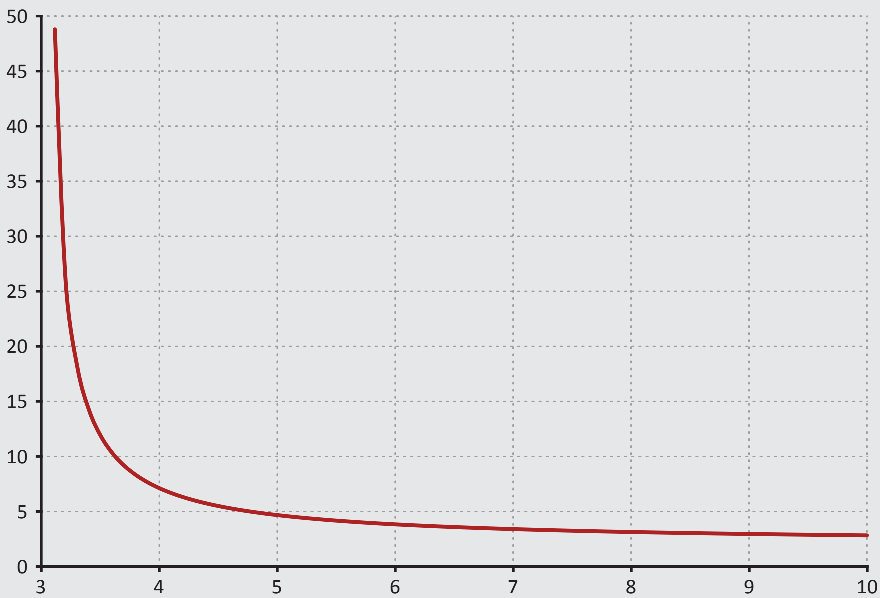
Figure 6.
Value of vcb_α as a function of parameter *a*



Source: Own calculations.

If $a \leq 1$, the expected value of the distribution is infinite, thus it makes sense to define the vcb_{α} only when a is greater than 1. By increasing a the tail of the distribution keeps getting thinner and less skewed to the right[20] (see *Figure 7*). Experiences show in the cases that are relevant for us (where $VaR_{\alpha}(X) > E(X)$) that by increasing a the capital buffer also increases under any fixed confidence level; thus it is true here as well that by increasing the skewness or making the tail of the distribution fatter the value of vcb_{α} decreases.

Figure 7.
Pearson's skewness of the Pareto distribution as a function of parameter *a (c=1)*



Source: Own calculations.

It has only theoretical relevance to examine the boundary value of vcb_{α}, when a converges to the infinite. It can be easily deduced *(21)*

$$\lim_{a\to\infty} vcb_{\alpha} = \frac{\ln(1-\alpha)+1}{\ln(1-99.5\%)+1}, \qquad (21)$$

which is positive only if $\alpha > 1-\frac{1}{e} \sim 63.2\%$, i.e. under a confidence level of 63.2 per cent, we get negative volatility capital buffer for all loss functions of Pareto distribution. Usually $VaR_{\alpha}(X)>E(X)$ is fulfilled, if $\alpha > 1-\left(1-\frac{1}{a}\right)^{a}$. The threshold α parameters belonging to the individual a parameters are explained in *Table 5.*

[20] It is easy to conceive that Pearson's skewness $\gamma = \frac{2(a+1)}{a-3}\sqrt{\frac{a-2}{a}}$ *(a>3)* decreases monotonously in a.

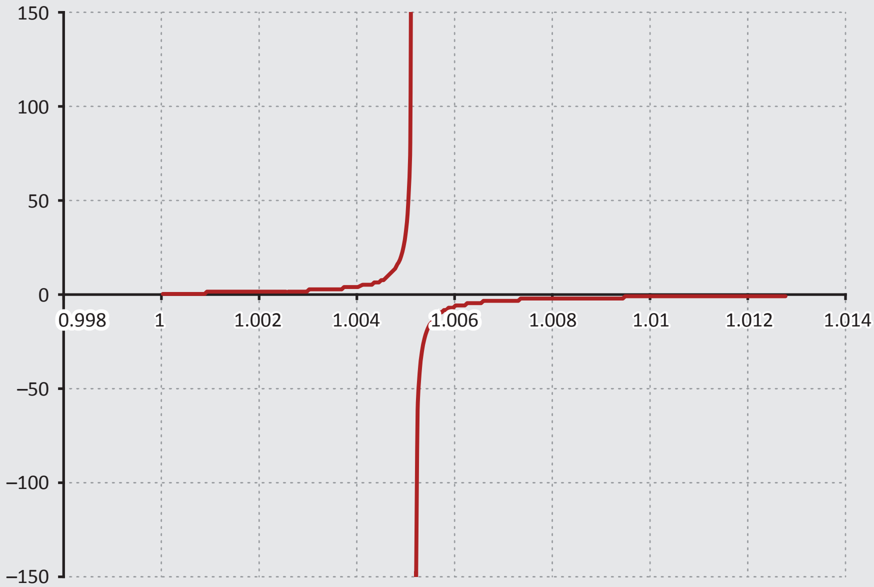
Table 5.
vcb_α values in the case of Pareto distribution under different *a* parameters

	a = 1,5	a = 2	a = 5	a = 10	a = 1000	a = 1E+09
60%	–3.7%	–3.4%	–3.0%	–2.6%	–2.0%	–1.9%
65%	–3.2%	–2.6%	–1.0%	–0.1%	1.1%	1.2%
75%	–1.5%	0.0%	4.3%	6.4%	9.0%	9.0%
85%	1.7%	4.8%	12.9%	16.6%	20.8%	20.9%
95%	14.0%	20.4%	34.9%	40.5%	46.4%	46.4%

Source: Own calculations.

In the case of the various distribution families, changing the parameters modifies not only the distance of VaR_α and $VaR_{99.5\%}$ from the expected value, but also the relation between them. If α < 99.5% then $VaR_\alpha < VaR_{99.5\%}$, but the expected value may be anywhere relative to these. For example, in the case of Pareto distribution, if we approximate to 1 with parameter *a*, all three values will increase, but it is the expected value that increases the fastest, "overtaking" first the VaR_α and the $VaR_{99.5\%}$ value. Thus $vcb_\alpha = \frac{(1-\alpha)^{-1/a} - \frac{a}{a-1}}{(1-0.995)^{-1/a} - \frac{a}{a-1}}$, as the function of parameter *a*, becomes negative under any *α < 99.5%* and first converges to the minus infinite, then – after a discontinuity – it converges from the plus infinite to 1 [21] (see Figure 8). That is, any

Figure 8.
Behaviour of vcb_α near *a = 1 (α = 75%)*



Source: Own calculations.

[21] It is not difficult to show the latter.

value may be obtained for the volatility capital buffer even if we assume a Pareto distribution (see also Section 4.2.1).

4.1.2.5. Gamma distribution

Gamma distribution probability density function *(22)*

$$f(x)=\frac{\lambda^{p}x^{p-1}e^{-\lambda x}}{\Gamma(p)}, \tag{22}$$

which in the case of $p=1$ corresponds to the probability density function of the exponential distribution. ($\Gamma(p)$ is the gamma function[22].) In the case of $p \leq 1$ $f(x)$ converges to the infinite, if x (from the positive side) converges to zero, and in the case of $p > 1$ it converges to zero. Increasing p will make the tail of the distribution thinner[23], and reduce the skewness, while the changing of λ means only a linear transformation, i.e. it is indifferent for us.

We also get a gamma distribution by the convolution[24] of p pieces of fully independent exponential distributions with parameter λ (*Newton L. Bowers et al. 1997*). As a result of the central limit theorem, the standardised gamma distribution[25] keeps approximating the standard normal distribution by increasing p. Thus – as vcb_α is invariant to the linear transformation of the distribution – it is not surprising that the vcb_α values obtained under p are very much similar to the figures obtained under the normal distribution.

Experiences show that the value of vcb_α increases under a fixed α, if p increases, i.e. it is true here as well that increasing of the skewness or making the tail of the distribution fatter reduces the value of the capital buffer.

Table 6.
vcb_α values in the case of gamma distribution under different p parameters

	0,5	1	1,5	4	10	1000	1E+09
65%	–1.8%	1.2%	2.9%	6.5%	9.1%	14.3%	15.0%
75%	4.7%	9.0%	11.3%	15.9%	19.1%	25.4%	26.2%
85%	15.6%	20.9%	23.6%	28.9%	32.5%	39.4%	40.2%
95%	41.3%	46.4%	48.9%	53.8%	57.1%	63.1%	63.9%

Source: Own calculations.

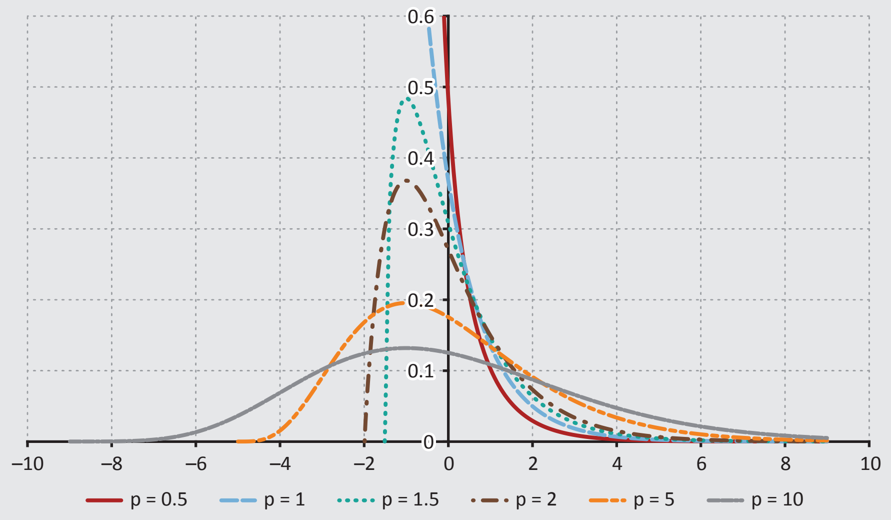
[22] $\Gamma(p)=\int_0^\infty t^{p-1}e^{-t}\,\mathrm{d}t$ expansion of the factorial function: *Γ(n)=(n-1)!,* if n is non-negative integer.

[23] Increasing of p makes the tail of the distribution thinner in the following sense: distribution *X* is of more fat-tailed than distribution *Y,* if for the probability density functions of their standardised version *f(x)* and *g(x),* respectively $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$.

[24] The distribution of the sum of the probability variables is the convolution of the individual distributions.

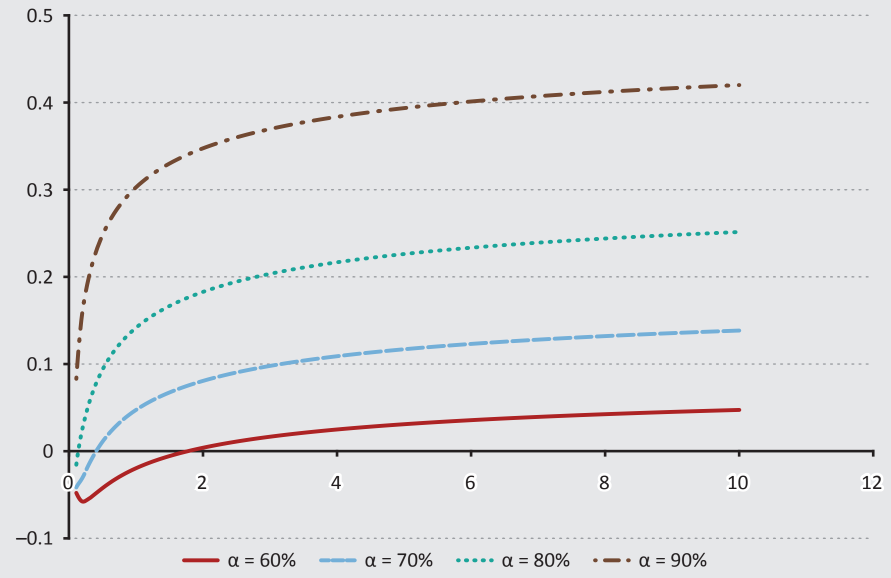
[25] The standardised version of the X probability variable is the linear transformed version of *X,* the expected value of which is zero, and its standard deviation is *1: X' = (X – E(X))/D(X),* where *E(X)* is the expected value, *D(X)* is the standard deviation, provided that these do exist.

Figure 9.
Probability density functions of the gamma distribution shifted to zero expected value, under different *p*'s *(λ = 1)*



Source: Own calculations.

Figure 10.
Value of vcb_{α} as a function of parameter *p*



Source: Own calculations.

4.1.2.6. Weibull distribution

The Weibull distribution is also the expansion of the exponential distribution. Probability density function *(23)*

$$f(x)=\frac{k}{\lambda}\cdot\left(\frac{x}{\lambda}\right)^{k-1}\cdot e^{-\left(\frac{x}{\lambda}\right)^{k}}\quad (x\geq 0,k,\lambda>0). \tag{23}$$

This may be used for modelling the time to breakdown (death). In the case of *k<1* with the passing of time it models decreasing (e.g. infant mortality), while in the case of *k>1* it models an end (e.g. car theft, old-age mortality) of increasing probability, and in the case of , *k=1* it models breakdown independent of time (e.g. electric bulb). The increasing of *k* reduces the skewness of the distribution, and makes the tail of the distribution thinner (see *footnote 23*).

The rate of the capital buffer *(24)* can be easily deduced here as well

$$vcb_{\alpha}=\frac{\left(-\ln(1-\alpha)\right)^{\frac{1}{k}}-\Gamma\left(1+\frac{1}{k}\right)}{\left(-\ln(1-99.5\%)\right)^{\frac{1}{k}}-\Gamma\left(1+\frac{1}{k}\right)}, \tag{24}$$

where *Γ* is the already mentioned gamma function. The obtained expression does not depend on *λ* due to the displacement invariance.

Experiences show that the vcb_α increases monotonously in the positive range, if we increase the value of *k*[26], i.e. it is true here as well that the higher skewness or the more fat-tailed distribution entails lower capital buffer. When converging *k* to *0* we obtain negative capital buffer even at the higher confidence levels. It is conceivable that if we increase the value of *k* beyond any limit, the limit of vcb_α is

$$\lim_{k\to\infty} vcb_{\alpha}=\frac{\ln\left(-\ln(1-\alpha)\right)-\gamma}{\ln\left(-\ln(1-99.5\%)\right)-\gamma}, \tag{25}$$

where *γ* is the Euler–Mascheroni gamma (~0.5772) (*Jeffrey C. Lagarias 2013*).

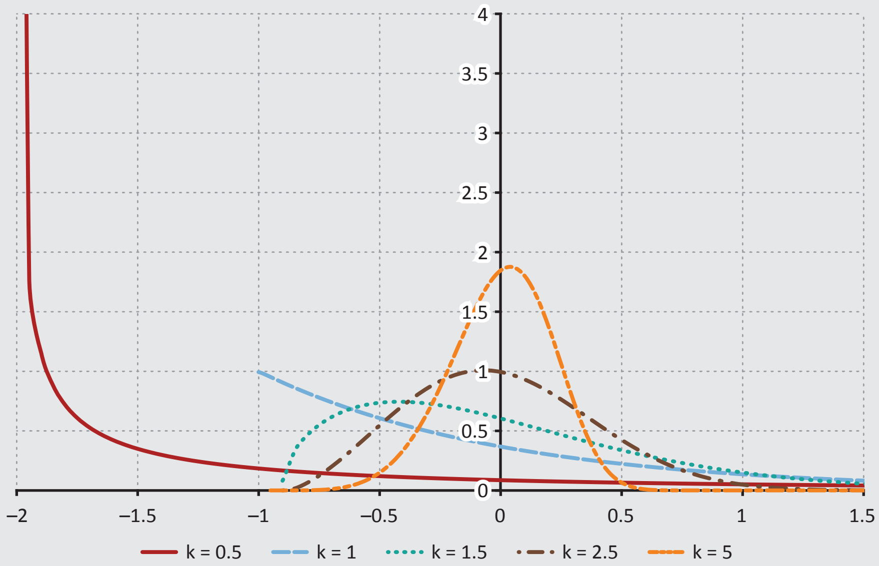
Table 7.
vcb_α values in the case of Weibull distribution under different k parameters

	0,5	1	1,5	2,5	5	1000
65%	–3.4%	1.2%	6.1%	12.5%	19.2%	27.8%
75%	–0.3%	9.0%	15.9%	23.8%	31.3%	40.2%
85%	6.1%	20.9%	29.5%	38.1%	45.7%	54.2%
95%	26.8%	46.4%	55.0%	62.6%	68.5%	74.6%

Source: Own calculations.

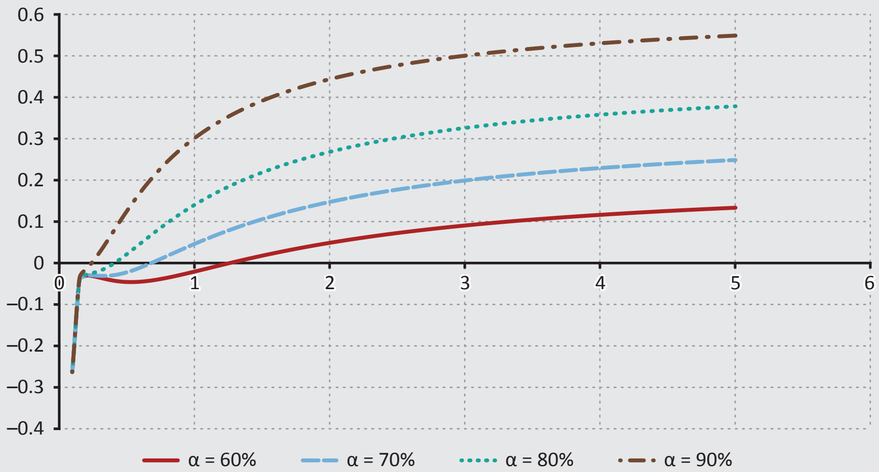
[26] If *α≥75%,* then experiences show that vcb_α keeps monotonously increasing in k. The statement has not been proven formally.

Figure 11.
Probability density functions of the Weibull distribution shifted to zero expected value, under different *k* parameters *(λ = 1)*



Source: Own calculations.

Figure 12.
Value of vcb_α as a function of parameter *k*



Source: Own calculations.

4.2. Estimations performed based on general distribution attributes

4.2.1. Arbitrary distribution

The real distribution of an insurer's loss does not belong to any of the distribution families; i.e. no matter which distribution family is assumed, it cannot be guaranteed that the capital buffer determined on the basis thereof guarantees the prescribed capital adequacy in the interim period at the targeted confidence level. Is it possible to find a universal p_upper_α or p_lower_α parameter that in the case of an arbitrary X distribution function with expected value of zero

$$p\_lower_\alpha \cdot VaR_{99.5\%}(X) \leq VaR_\alpha(X) \leq p\_upper_\alpha \cdot VaR_{99.5\%}(X)? \qquad (26)$$

It was presented in Section 4.2.2.4 that such universal parameters do not exist even if we assume in respect of X that it is of Pareto distribution. However, here for the really interesting case – when the expected value of X is higher than both $VaR_\alpha(X)$ and $VaR_{99.5\%}(X)$ – we obtained an absolute upper bound of $\frac{\ln(1-\alpha)+1}{\ln(1-99.5\%)+1}$.

The simple examples below show that if we assume nothing in respect of X, then the value of the quotient (27)

$$\frac{VaR_\alpha(X)}{VaR_{99.5\%}(X)} = \frac{VaR_\alpha(X)}{SRC} \qquad (27)$$

can be anything.

Let us examine the following distribution family, the (28) probability density function of which[27]:

$$f(x) = \begin{cases} A, if \ \ 0 \leq x < 1 \\ B, if \ \ 1 \leq x \leq b+1 \\ 0 \ otherwise \end{cases} \qquad (28)$$

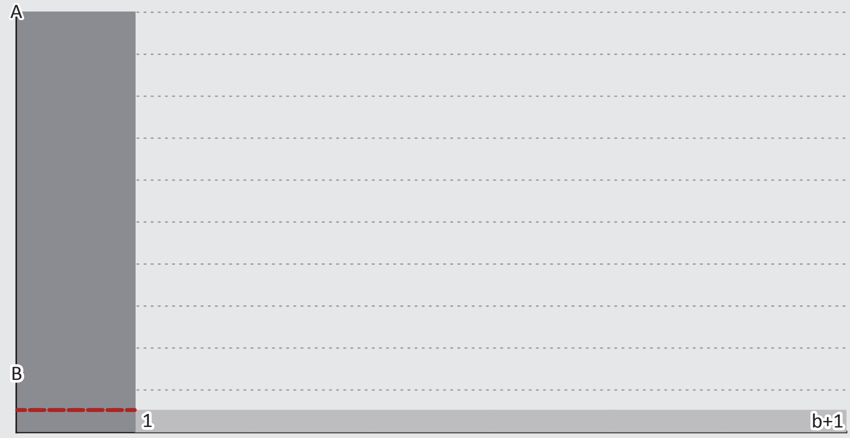
Under $A = 0.995$, $B = 0.0000125$, $b = 400$ the expected value of X is $E(X) = 1,5$, while its 99.5 per cent quantile will be $VaR_{99.5\%}(X) = 1$. If the loss of an insurer were of such distribution, it would mean that the 99.5 per cent quantile of its unexpected loss, i.e. its solvency capital requirement would be $SCR = VaR_{99.5\%}(X) - E(X) = -0,5$, i.e. negative. If under a fixed $B = 0.0000125$ A is gradually decreasing to the critical 0.99499368712625 value (and in parallel with this we increase b such that the $f(x)$ remains a probability density function[28]), the $VaR_{99.5\%}(X) – E(X)$ gradually converges to zero from the negative side, while under arbitrary $\alpha < 99$ per cent confidence the $VaR_\alpha(X)$ will be always between -1.5 and -0.5, that is *(29)*

$$\lim \frac{Var_\alpha(X) - E(X)}{SCR - E(X)} = +\infty \qquad (29)$$

[27] We could build a portfolio that has similar distribution and thus the aforementioned circumstances would fit the insurer thus created, but in real life no such loss distribution occurs.

[28] That is, the field below f(x), in our case should be A*1+b*B = 1.

Figure 13.
Probability density function of the distribution presented in the sample



Due to similar considerations the quotient converges to the minus infinite, if A is gradually increasing toward the critical 0.99499368712625 value. Thus, the vcb_{α} may take any value.

Even then, we can only provide for the searched quotient the (30) trivial

$$0 \leq \frac{VaR_{\alpha}(X)}{VaR_{99.5\%}(X)} = \frac{VaR_{\alpha}(X)}{SCR} \leq 1 \tag{30}$$

estimation, if we assume that both the SCR and the VaR are positive: Setting out from the above distribution, for the arbitrary α we can find such A (and the corresponding b) that ensures that $VaR_{\alpha}(X) - E(X)= 0$, i.e. usually it is not possible to provide a better estimate than the left side of the above trivial estimate. Keeping the same example, let us fix the value of A as $A = \alpha$! If we converge with B to the plus infinite (and simultaneously modify b to ensure that $A + b*B = 1$ is maintained), then the $\frac{VaR_{\alpha}(X)}{SCR}$ quotient converges to 1, i.e. usually it is not possible to find a better estimation than the right side of the trivial inequality, other assumptions must be made with regard to the distribution of the unexpected loss.

4.2.2. Unexpected losses with decreasing probability

It is a natural assumption that the probability of the unexpected loss decreases with the degree of the loss; or to put it more accurately: $P(a<X<a+\varepsilon) \leq P(b<X<b+\varepsilon)$, if $a \geq b > 0$, where X is the unexpected loss, ε is an arbitrary positive number. If distribution X has probability density function f, then this condition is equivalent to f decreasing monotonously in the *[0;∞)* interval. It is not difficult to show that then in case of α<99,5% (31)

$$\frac{VaR_{\alpha}(X)}{VaR_{99.5\%}(X)}=\frac{VaR_{\alpha}(X)}{SCR}\leq\frac{\alpha-p}{99.5\%-p}, \tag{31}$$

where $p=P(X<0)=\int_{\infty}^{0} f$. If X is of symmetric distribution (i.e. f is an even function), then $p = 0.5$. The key argument against the assumption of normal distribution is that the value of the unexpected loss usually exceeds the rate of the unexpected profit, the average of the unexpected losses is typically higher than the average of the unexpected profits.

Let us denote the average of the unexpected losses *(32)* with V!

$$V=E(X|X>0) \tag{32}$$

where $E(X|X>0)$ means the conditional expected value of X under the condition of $X>0$. Let us denote the unexpected profits *(33)* with N!

$$N=E(X|X<0) \tag{33}$$

Considering that $E(X) = 0$, i.e. $E(X^+)=-E(X^-)$, where $X^+=max(X;0)$, $X^-=min(X;0)$, and that it is followed by $E(X|X>0)=E(X^+)/P(X>0)$ and $E(X|X<0)=E(X^-)/P(X<0)$ (34)

$$\frac{p}{1-p}=\frac{V}{N}, \tag{34}$$

where $p = P(X<0)$ in accordance with the foregoing. If the average unexpected loss is higher than the average unexpected profit, i.e. $V>N$, then $p>0.5$. As the $\frac{\alpha-p}{99.5\%-p}$ expression decreases monotonously in p, the $V>N$ is fulfilled in the case of (35) for all $\alpha < 99.5\%$ confidence levels.

$$\frac{VaR_{\alpha}(X)}{VaR_{99.5\%}(X)}=\frac{VaR_{\alpha}(X)}{SCR}\leq\frac{\alpha-0.5}{99.5\%-0.5} \tag{35}$$

Consequently, for example $vcb_{75\%}\leq 0,505$, i.e. in the case of a 150.5 per cent capital level each insurance company whose unexpected loss distribution satisfies the following two conditions will comply with the capital requirements with a probability of at least 75 per cent: *(i)* the probability of the larger unexpected losses is lower; *(ii)* the unexpected losses on average are higher than the unexpected profits.

However, if we know, for example, that the average of the unexpected losses is twice as high as that of the unexpected profits (then $p \geq 2/3$) then even a capital level of 125.4 per cent is sufficient, with a probability of 75 per cent, for the capital adequacy on a one-year horizon.

Table 8.
Upper estimate for the value of vcb_α as a function of the quotient of the average of unexpected losses and unexpected profits *(V/N)*

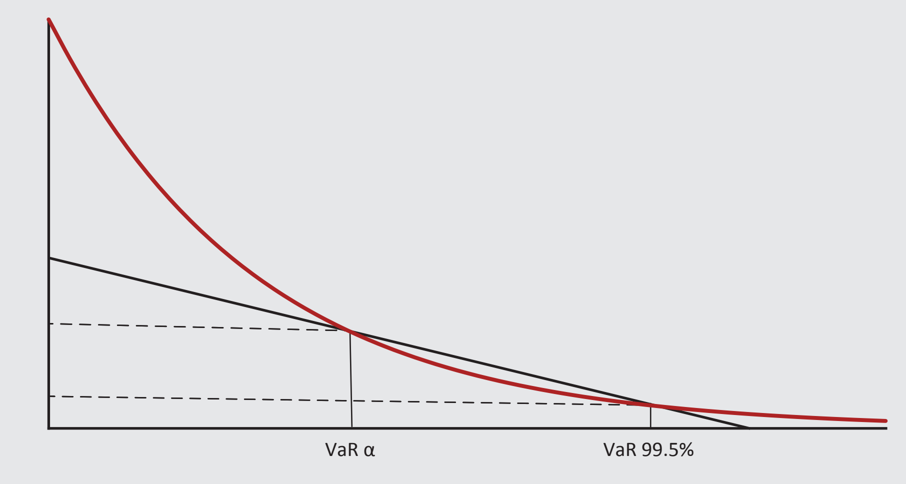
	1	1,2	1,5	2	4	10
65%	30.3%	23.3%	12.7%	–5.1%	–76.9%	–301.6%
75%	50.5%	45.5%	38.0%	25.4%	–25.6%	–185.2%
85%	70.7%	67.7%	63.3%	55.8%	25.6%	–68.8%
95%	90.9%	90.0%	88.6%	86.3%	76.9%	47.6%

Source: Own calculations.

4.2.3. Unexpected losses of a probability that declines at a decreasing rate

In the previous Section, we used the natural assumption that the probability of increasing losses keeps decreasing, that is the *f(x)* probability density function of the unexpected loss is monotonously decreasing, if $x>0$ $(E(X) = 0)$. It can be assumed especially in the case of the fatter-tailed and strongly right-skewed distributions that although the probability of increasing losses keeps decreasing, the intensity of the decrease is also becoming lower and lower, i.e. the probability density function is monotonously decreasing and convex in the case of $x>0$ ($E(X) = 0$).

Figure 14.
Probability density function of the distribution presented in the sample



Using the fact that the secant drawn for the probability density function's curve for the α and 99.5 per cent quantiles runs under the probability density function, if $0<x<VaR_\alpha$ and if $VaR_{99.5\%}<X$, and over, if $VaR_\alpha < x < VaR_{99.5\%}$ (see Figure 14), we get the following (36) inequality:

$$vcb_\alpha = \frac{VaR_\alpha}{VaR_{99.5\%}} \geq \frac{\sqrt{1-p} - \sqrt{1-\alpha}}{\sqrt{1-p} - \sqrt{1-99.5\%}}, \quad \text{where} \tag{36}$$

in accordance with the foregoing $p = P(X < 0) = \int_{-\infty}^{0} f$.

In a symmetrical case, or if the average of unexpected losses equals one of unexpected profits (i.e. *p = 0.5*), the minimum capital level necessary for the 75 per cent confidence level is 132.5 per cent. However, a *V/N* ratio higher than one better fits the assumptions made at the beginning of the section. For example, in the case of *V/N = 2*, also mentioned in the previous section, the minimum capital level required for a 75 per cent confidence is 115.3 per cent. If we also rely on the results of the previous section, the target capital level is somewhere between 115.3 and 125.4 per cent in this case.

Table 9.
Lower estimate for the value of vcb_α as a function of the quotient of the average of unexpected losses and unexpected profits *(V/N)*

V/N	1	1,2	1,5	2	4	10
65%	18.1%	13.7%	7.3%	–2.8%	–38.4%	–125.7%
75%	32.5%	28.9%	23.6%	15.3%	–14.0%	–86.0%
85%	50.3%	47.5%	43.6%	37.5%	15.9%	–37.2%
95%	76.0%	74.7%	72.8%	69.8%	59.4%	33.8%

Source: Own calculations.

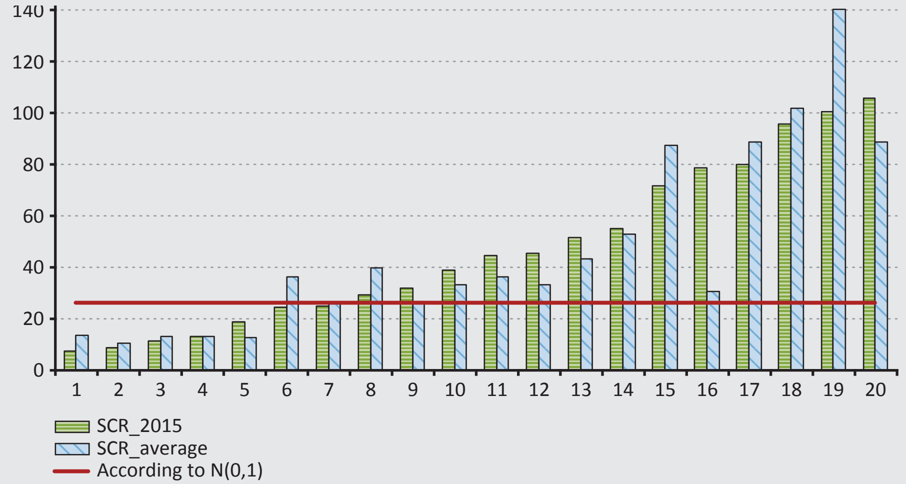
4.3. Estimation performed on the basis of empirical variance

Based on the considerations outlined in Section 1, on a one-year horizon the volatility capital buffer at the given α confidence level should guarantee that the insurer's solvency capital would not decrease below the level of the last determined solvency capital requirement due to unexpected losses incurred on its existing portfolio and the portfolio to be acquired in the next 12 months as a result of environmental changes. Apart from the difficulties related to the empirical approach listed in Section 3.3, one of the biggest problems is that we have no data for the capital decrease attributable specifically to the above mentioned circumstances and only estimates can be performed for this in the future as well.

Of the available data, it is the change in the net asset value that is the closest to the volume to be examined based on the foregoing, which has to be adjusted for the external capital flows (capital injection, dividends). For this we may use the data of the QIS5 (*2010*), QIS5bis (*2011*), and the 2012 and 2014 impact studies, and the RIGL preliminary Solvency II data supply (*2015*). Only those insurers should be involved in the comparison that participated in at least four of the above five data supplies, although the standard deviation cannot be estimated reliably even in the case of a sample with four or five elements. The result should be further adjusted for the expected profit/loss of the new portfolio, but this is not possible. In certain cases the volatility may have been considerably influenced by the fact that the insurers provide the data on "best effort" basis, and thus the data did not fully cover the objectives outlined by the Solvency II regime.

In accordance with Section 3.3, we may apply the 2/3 rule for the 75 per cent confidence level, based on which the empirical volatility capital buffer is 2/3 of the standard deviation of the net asset values adjusted in accordance with the previous paragraph. If the value of the empirical capital buffer is converted into the targeted capital level, i.e. divide it by the SCR value – assuming normal distribution – we should obtain additional capital levels around 26.2 per cent.

Figure 15.
Empirical volatility capital buffer as a percentage of SCR



Source: Own calculations.

However, the results obtained from the foregoing should be viewed critically. It is not surprising that at 65 per cent of the 20 insurers, that may be considered on the basis of the empirical data, the capital buffer is higher than that based on the hypothetical model assuming normal distribution (which usually results in upper estimates) and in the case of 35 per cent of them the difference is more than twofold.

In view of the fact that the empirical data belong to different periods and different statuses, it is worth performing the comparison (conversion) also with the average of the previous SCR's; however, the difference between the empirical and hypothetical approach hardly decreases here as well.

5. Summary

The volatility capital buffer – as capital held in addition to the capital requirement – is meant to reduce the risk that the insurer's solvency capital falls below the last determined and reported capital requirement in the interim period when the insurer does not calculate its solvency capital. This paper details an approach with which the task can be reduced to searching for the quantile belonging to a given confidence level of the same probability variable, which is the 99.5 per cent quantile of the solvency capital requirement (SCR).

Taking the SCR standard formula as a basis, with this approach the VCB reduces the insurer's risk of capital shortfall in the insurer's basic own funds (net asset value) arising from the *unexpected loss* incurred on the existing portfolio and the portfolio to be acquired in the next 12 months. The surplus capital requirement thus obtained may be significantly overruled, for example, by the profit expected to be realised from the expected renewal of the insurer's existing contract portfolio.

It follows from the approach that the value of the VCB is proportionate to the SCR. The $vcb = \frac{VCB}{SRC}$ ratio thus obtained may have any value, if we apply no restrictions for the distribution of the unexpected loss. Assuming normal distribution, the VCB ratio belonging to the 75 per cent and 90 per cent confidence level is 26.2 and 49.8 per cent, respectively. In the case of the distribution families used for modelling the claim distributions, the VCB value may depend (occasionally – e.g. Parato – to an extreme degree) on the distribution parameters: the capital buffer rate decreases with the fattening of the distribution's tail and right skewness.

The value of the capital buffer may be defined within a narrow range under certain reasonable general assumptions. Assuming that the probability density function of the distribution is monotonously decreasing and it is convex in the *(0; ∞)* interval, then depending on the ratio of the expected loss value (*V*) and the expected profit value (*E*), we can provide lower and upper estimates, being close to each other, for the *VCB*. For example, in the case of a 75% per cent confidence level under a *V/E = 1* ratio the VCB falls between 32.5 and 50.5 per cent , while under *V/E = 2* it is between 15.3 and 25.4 per cent.

The above analyses and calculations show that the capital buffer to be targeted by the insurers and expected by the supervisory authority may be fundamentally influenced – among others – by the assumed distribution, the targeted confidence level and the consideration of the expected profit/loss on the future portfolio, but upon defining the ultimate rate other considerations may also emerge (e.g. prudency, simplicity).

References

Arató, M. (1995): *General actuarial mathematics*, lecture notes, pp. 33–52

Azzalini1: *A very brief introduction to the skew-normal distribution*. http://azzalini.stat.unipd.it/SN/Intro/intro.html. Downloaded: 11 September 2015

Azzalini2: *Random numbers with SN or ST distribution*. http://azzalini.stat.unipd.it/SN/faq-r.html. Downloaded: 11 September 2015

Bora, Zs. – Engler, K. – Holczinger, N. – Jakab, K. – Merész, G. – Nagy, K. – Zubor, Z. (2015): *What does Solvency II bring for the Hungarian insurance sector?* Biztosítás és Kockázat (Insurance and Risk) Volume 2, Issue 1, pp. 51–52.

Newton L. Bowers, jr. - Hans U. Gerber – James C. Hickman – Donald A. Jones – Cecil J. Nesbitt (1997): *Actuarial Mathematics,* The Society of Actuaries, 1997, pp. 377.

Insurance Europe (2013): *The package of measures to avoid artificial volatility and pro-cyclicality*. http://www.insuranceeurope.eu/sites/default/files/attachments/The%20package%20of%20measures%20to%20avoid%20artificial%20volatility%20and%20pro-cyclicality.pdf. Downloaded: 19 January 2016

Jeffrey C. Lagarias (2013): *Euler's Constant: Euler's Work and Modern Developments*. Bulletin (New Series) of the American Mathematical Society, Volume 50, Number 4, October 2013, pp. 527–628 Article electronically published on July 19, 2013, http://www.ams.org/journals/bull/2013-50-04/S0273-0979-2013-01423-X/S0273-0979-2013-01423-X.pdf. Downloaded: 11 September 2015

EIOPA (2011): *EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II*, pp. 26. https://eiopa.europa.eu/Publications/Reports/QIS5_Report_Final.pdf. Downloaded: 19 January 2016

EIOPA (2013): *Technical Findings on the Long-Term Guarantees Assessment*, pp 21-22. https://eiopa.europa.eu /Publications/QIS/EIOPA_LTGA_Report_ 14_June_2013_01.pdf. Downloaded: 19 January 2016

EIOPA (2014): *EIOPA Insurance stress test 2014*, pp. 51-52 https://eiopa.europa.eu/Publications/Surveys/ Stress%20Test%20Report%202014.pdf. Downloaded: 19 January 2016

MNB (2015a): *Financial Stability Report May 2015*, pp. 69 https://www.mnb.hu/letoltes/penzugyi-stabilitasi-jelentes-2015-majus.pdf. Downloaded: 19 January 2016

MNB (2015b): *Risk outlook for non-bank financial sectors, June 2015*, pp. 33-34 https://www.mnb.hu/letoltes/bankszektoron-kivuli-penzugyi-piacok-kockazati-jelentese-2015-junius-1.pdf. Downloaded: 19 January 2016