Topology of the foreign currency/forint swap market*

Ádám Banai – András Kollarik – András Szabó-Solticzky

In our study, we examine the network structure of the currency swap market, the volume of which amounts to several times the Hungarian GDP. With this paper, we aim to establish a complete picture of the market by complementing the results for the overnight market. Additionally, we can now analyse a longer time series than in our 2013 study. We look at the properties of the graphs in segments representing various maturities. We find that the properties of the graph derived from the overall market and the dynamics of those properties are identical to those of the short-term market, while trends differ for various tenors. The longer the maturities, the less the graphs satisfy the small-world property. The longest markets are increasingly closer to random graphs. Although the effect of shocks to large actors is smaller in such graphs, this change also suggests that counterparties trusted each other less as transactions became longer. This is also reinforced by the fact that following the onset of the crisis, the number of connected vertices gradually decreased in the networks of longer markets. In other words, weakening trust is also manifested in the decreasing number of counterparties. This is confirmed by the development of average degree and average path length, and by affinity functions.

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1. Introduction and precedents in literature

The global economic crisis which erupted in 2007 hit Hungary severely, as the financial markets collapsed following the Lehman bankruptcy. In October 2008, a few weeks after the Lehman default, the key Hungarian money markets also froze up. The government securities market, the unsecured interbank forint market, and the currency swap market all came to a standstill for a few days, which hit the banking system severely as well. Banks are required to continuously renew their

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Ádám Banai is Head of Applied Research and Stress Testing Department at the Magyar Nemzeti Bank. E-mail: banaia@mnb.hu.

András Kollarik is an economic analyst of the Monetary Policy Instruments and Reserve Strategy Department at the Magyar Nemzeti Bank. E-mail: kollarika@mnb.hu.

András Szabó-Solticzky is a PhD candidate in Applied Analysis and Computational Mathematics at Loránd Eötvös Science University. E-mail: szabosolticzky@gmail.com.

currency swaps, in order to close their on-balance sheet open foreign exchange positions off-balance sheet. If they had not been able to renew their expiring swaps, to meet their liabilities they would have been forced to purchase currency on the spot market, which would have placed the forint exchange rate under enormous pressure. In dealing with the problems on the swap market, a major role was played by parent bank commitment, and it was also necessary for the Magyar Nemzeti Bank to introduce new swap instruments.

This episode of the crisis highlighted the key importance of the operation of the FX swap market for financial intermediation in Hungary. Hungarian banks gain access to foreign currency funds partly through FX swaps, which are essential for closing their open positions resulting from their significant portfolios of foreign currency loans. Foreign actors are also particularly active in this market. They enter forward positions through FX swaps, which in many cases they also use to hedge the exchange rate risk of their forint assets. The FX swap market also deserves particular attention from a monetary policy perspective. Disruptions to the operation of this market may significantly reduce forint implied yields, making speculation against the forint cheaper.

The operation and role of the Hungarian currency swap market has been addressed by a number of studies. *Páles et al. (2010)* provide a highly detailed description of the role played by the market in Hungary's economy. They show how different the motivations of various actors (domestic banks, foreign actors) are for entering the FX swap market. The authors also give an overview of the changes which occurred in the market during the crisis. In the aftermath of the turmoil, in many cases limits for domestic banks were reduced, accompanied by an increased role of margin calls, which further intensified the demand for swaps. *Banai et al. (2010)* also address the problems experienced in the FX swap market during the crisis. They find that, prior to the escalation of the crisis in Hungary, it was local banks in particular that had accumulated extremely large stocks of FX swaps. Stock renewal and increasing margining posed a major problem at certain stages of the crisis, which called for the use of the MNB's instruments.

Analyses of the Hungarian FX swap market have mainly focused on the stock size and maturity, the development of implied yields, and the behaviours and strategies of the various actors. However, additional information about financial markets may be obtained through an exploration of the actors' network relationships. Research on the network characteristics of financial markets has gained prominence in the past decade, especially since the onset of the crisis. One of the first studies of this kind was *Lublóy (2006)*, which discussed the network structure of VIBER. According to the study, the network characteristics under review were stable in time. Additionally, the author identified the actors that were the most important for the stability of the network. Surprisingly, these institutions were not the largest banks of the banking system in terms of balance sheet total. In this paper, we rely heavily on our previous study on the short-term FX swap market (*Banai et al. 2013*). In our analysis, we demonstrated that the overnight FX swap market also shares the small-world property characteristic of financial networks (for details, see Chapter 4 on methodology). Additionally, an examination of the dynamics of network indicators highlighted the significant decrease in the size of the short-term FX swap market after the Lehman bankruptcy. That is, many institutions either cut down on their activity in the market, or left it entirely. However, the exiting institutions were predominantly those with less relevance to the market. This is definitely beneficial in terms of stability. In turn, there is a greater risk from the increased role of the remaining institutions, i.e. the fact that the market has become more sensitive to the behaviour of particular actors.

The tools of network theory are used even more frequently in international literature to examine financial markets. In their research, lazzetta and Manna (2009) addressed the properties of the Italian interbank market. They explored the behaviour of a number of network characteristics. They found the connectivity of the network to be very low, similarly to real networks, and to be decreasing in time. Another key observation was that the entire network remained connected throughout the 222-month period under review. That is, with any pair of banks, it was possible to identify a path through which they could reach each other. Their third finding was that, although to a minor extent, the average shortest path increased within the network (for details, see Chapter 4 on methodology). They also found that the largest actors were dealing regularly and directly with counterparties playing smaller roles in the network. Finally, the authors demonstrated that the proportion of relatively large actors in the network had decreased, accompanied by an increase in the number of banks dealing only with a few counterparties. In their study, lori et al. (2008) discussed the Italian overnight market. They demonstrated that banks that were large in terms of degree had a great number of small counterparties. The authors found this to increase the risks of contagion in the case of high-density networks (see details in Chapter 4 on methodology).

Soramäki et al. (2006) analysed the network properties of interbank payment data in the Fedwire Funds Service. They found this network also to be sharing the properties characteristic of real networks. Such features include scale-free degree distribution, a high clustering coefficient, and the small-world phenomenon, which was introduced by *Watts and Strogatz (1998)* and is generally characteristic of financial networks. They also found the properties of the networks under review to be stable in time.

The above list is sufficient to show that in finance, network tools have primarily been used to analyse payment and settlement systems as well as unsecured interbank markets (although *Markose et al. (2010)* discussed the US CDS market). In neither the Hungarian nor the international literature have we found any papers providing

a network analysis of the FX swap market. What follows is our attempt at such an analysis. As we previously discussed the processes of the short-term market in detail (*Banai et al. 2013*), in this study our primary focus will be on longer markets and their relationship to the short-term market. In Chapter 2, we provide a more detailed description of the main characteristics of the Hungarian currency swap market. In Chapter 3, we discuss the data used and in Chapter 4, we describe the network theory methodology applied. Then, in Chapter 5, we present our results on the topology of the network. Finally, in Chapter 6, we summarise our findings.

2. The currency swap market

2.1. The FX swap

A foreign exchange swap (hereinafter: FX swap) is a derivative financial transaction consisting of two legs. On the start (or spot) leg of the transaction, the counterparties swap two different currencies with each other, which they will swap back on the maturing (or forward) leg. When entering into the transaction, they agree on both the spot and the forward exchange rate. FX swaps are also commonly called simple currency swaps, since no transaction takes place between the counterparties other than swapping nominal values, as opposed to currency interest rate swaps (CIRS), where the parties (also) pay interest to each other during the term. With FX swaps against the forint, the market convention is that the same foreign currency amount is traded on the spot and forward legs, and the two forint amounts payable are obtained as the forint value of that amount taken at spot and forward rates (*Figure 1*). The current exposure to a counterparty resulting from an



FX swap transaction as a secured loan (the net present value of the transaction) is an order of magnitude lower than that resulting from an unsecured deposit or loan transaction of the same nominal value.

FX swaps are used for various purposes in financial markets.

- 1. One widespread trading strategy involves an FX swap plus the purchase of a foreign currency asset (or the repayment of a foreign currency liability). In this case, the actor uses the liquidity acquired on the spot leg of the FX swap to purchase the foreign currency asset to be held (or to repay the foreign currency liability to be repaid), while the forward leg of the swap provides a hedge against the exchange rate risk of the foreign currency asset (or the foreign currency liability being repaid). Where the asset purchased and the swap mature at different times, the strategy also involves yield speculation. Foreign investors have a propensity to finance their forint government security purchases using FX swaps.
- 2. Another popular trading strategy is an FX swap and a foreign exchange spot market transaction of the opposite direction. This is equivalent to taking a forward foreign exchange position, since the spot transaction neutralises the spot leg of the swap, leaving only the forward leg effective. An actor may follow this strategy for both speculative (carry trade) and hedging purposes.
- 3. The third important strategy involves entry into two swaps of opposite directions, with the same spot value dates, but with different maturities. The strategy allows an actor to speculate on interest rate differentials. For instance, if they lend foreign currency in exchange for forints on the spot leg of the long swap, which is swapped back to foreign currency by a bank on the spot leg of a short swap of the opposite direction, then they may refinance the currency repayment upon maturity of the short swap through another short currency raising swap. In this case, profits will be generated for the bank if future short forint yields rise faster in relation to foreign currency yields than what is priced into long yields.
- 4. Another important role of FX swaps is to revolve maturing forward and FX swap transactions: the foreign currency funds acquired on the spot leg of the swap can be used to repay the foreign currency liability due, while the forward leg can be used to renew the off-balance sheet foreign currency debt.
- 5. Additionally, this type of transaction can be used solely to grant loans secured by the opposite currency.

2.2. The domestic currency swap market

Due to the limited data available, we will only analyse the segment of the domestic currency swap market where at least one of the counterparties to the transaction is a domestic bank. We have nothing more than anecdotal evidence that in London, foreign actors also enter into foreign currency/forint swap transactions (*Balogh–Gábriel 2003*). The FX swap market is a less strictly regulated OTC (over-the-counter) market, where foreign currency/forint transactions are typically traded through brokers in London, which means that direct bilateral contacts and market making are not common (*Csávás et al. 2006*). At the same time, in the segment of less than one month, by virtue of exclusive access to the MNB's forint market instruments, domestic banks may be considered to be market makers. *Table 1* provides a summary of the key properties of the market.

Table 1. Key characteristics of the domestic currency swap* market	
Indicator	Value
swap turnover among CHF, EUR and USD relative to HUF/FX swap turnover (2005-Nov. 2014)	44%
daily average turnover of HUF/FX swap market (2005-Nov. 2014, HUF bn)	525
gross HUF/FX swap stock of Hungarian banking system against foreigners (both directions, 1 Dec. 2014, HUF bn)	6006
ratio of interbank transactions to total turnover (2005-Nov. 2014)	95%
ratio of cross-border transactions to total turnover (2005-Nov. 2014)	84%
average transaction size (2005-Nov. 2014, HUF mn)	5474

*Currency swaps are understood in the broad sense, so we have taken into account FX swaps as well as CIRSs for our calculations. By default, we are describing the foreign currency/HUF market. Source: Own calculations based on MNB data.

Given the very high ratio of the volume of swaps between cross-currencies and in the foreign currency/forint segment, which points to the possibility of free movement from one foreign currency to another through swaps, the swap markets of various currencies against the forint are treated collectively as a general foreign currency/forint in the rest of this paper, without making a distinction between the USD/HUF and the EUR/HUF segments, for example.

The annual volume of the foreign currency/forint swap market amounted to approximately 5 times the Hungarian GDP in the sample period between 1 January 2005 and 1 December 2014. At the end of the period, the gross foreign currency/ forint swap stock of the Hungarian banking system against foreign actors, despite the continuous decrease of the stock of foreign currency loans, amounted to 18% of the balance sheet total of the banking system. These two figures also point to the significant role of the swap market in the domestic economy. Underlying this, in addition to the versatile applications of the transaction, there are macroeconomic

factors as well. Prior to the financial crisis starting on 15 September 2008 with the failure of Lehman Brothers, the country's net external debt and simultaneously the reliance of the banking system on foreign funds had increased significantly. Net external debt equals the sum of the long open forint foreign exchange positions taken by each economic sector (*Páles et al. 2010*), which means that the open forint position had to be taken by one economic actor or another. Foreign actors were less willing to do so, and therefore, most of the position was undertaken by the domestic private sector through the balance sheet of the banking system, which opened banks' on-balance sheet foreign exchange positions. Pursuant to an earlier government decree and the CRR, however, the banking system had to allocate capital to its entire foreign exchange position,¹ which gave it incentives to close the open position off-balance sheet. Typically, banks close their open on-balance sheet foreign exchange positions through currency swaps, using strategies 2 and 4 above (*Páles et al. 2010*).

2.3. The financial crisis and its effect on the currency swap market

Although the costs of funding in Hungarian markets had been increasing from mid-2007 as the subprime mortgage credit crisis unfolded, and a brief turbulence evolved in the government securities market in March 2008, the global financial crisis essentially escalated in Hungary following the collapse of Lehman Brothers on 15 September 2008. The most intensive phase of the crisis lasted from autumn 2008 to spring 2009. In the autumn of 2008, the cost of liquidity skyrocketed first in foreign currencies and then in forint as market players restricted their partner limits vis-à-vis one another. The turnover of the currency swap market became rather volatile, and the share of intra-group transactions also became unstable. While previously the maturity of swaps had gradually increased, this trend came to a standstill during the crisis, and new swaps became perceivably shorter. Additionally, the euro also replaced the dollar, which had previously played the dominant role in the swap market (*Páles et al. 2010*).

The overnight market dried up, and the aggregate liquidity index measuring market liquidity fell to – 8 by the end of October 2008 (*Figure 2*). This means that market liquidity was 8 standard deviations below the long-term average of the period preceding the crisis.² Meanwhile, swap spreads increased considerably. The spreads of around zero measured before the crisis widened to several hundred basis points. Consequently, in this period forint loans (secured by foreign currency) were available through swaps at rates several percentage points below the reference money market yield.

¹ Where the entire foreign exchange position exceeds 2% of total own funds, the own funds requirement for foreign exchange risk is 8% of the open foreign exchange position.

² For more details on the aggregate liquidity index, see Páles–Varga (2008).



Note: The liquidity index for the HUF swap market contains data on the one-day USD/HUF and EUR/HUF segments, where the maximum difference between the trade date and maturity is two business days. In every case, the greater value indicates an improvement in the given aspect of liquidity. Specific sub-indices are standardised by their pre-crisis long-term averages and standard deviations. Source: Own calculations based on data of the MNB, Bloomberg and Reuters.

3. Data

We calculated individual network indicators for longer segments (maturities of 3 days to 1 month, 1 to 3 months, and above 3 months), as well as for the graph derived from the overall market. The study will focus on the development of these (*Tables 2 and 3*). Due to its significant volume weight, in our analysis we also consider the 1–2 day segment as defined above. We do so because the above definition is consistent with the segment, the development of which is described by the aggregate liquidity index. However, while the aggregate liquidity index is calculated by the central bank only for the segment comprised of USD/HUF and EUR/HUF transactions, we also considered CHF/HUF transactions.

Table 2. Volume of the foreign currency/forint s	wap market	by currence	/	
	USD	EUR	CHF	together
proportion in turnover (%)	83	15	2	100
proportion in turnover corrected by tenor (%)	38	49	13	100
Note: Based on the farint value of the snat lea of	the transactio	nc. In adjustn	ants for mati	urity we mul

Note: Based on the forint value of the spot leg of the transactions. In adjustments for maturity, we multiplied the transaction value by the maturity. Source: Own calculations based on MNB data.

Table 3.

Volume of the foreign currency/forint swap market by maturity

	1-2 days	3 days-1 month	1-3 months	>3 months	together
proportion in turnover (%)	76	14	6	4	100
proportion in turnover corrected by tenor (%)	13	8	11	69	100

Note: Based on the forint value of the spot leg of the transactions. In adjustments for maturity, we multiplied the transaction value by the maturity. Source: Own calculations based on MNB data.

We examined the transactions completed between 1 January 2005 and 1 December 2014. The actors selected to represent the vertices of the graphs included both domestic and international actors, whereas the MNB was excluded. Our analysis concerned only credit institutions, as a result of which we ignored transactions with the non-financial corporate sector, for instance. Domestic banking groups were included in the graphs on a consolidated basis, i.e. members of banking groups were represented by a single vertex standing for the entire banking group. By contrast, we had no means to consolidate all of the foreign banking groups, neither did we intend to cleanse the database from transactions within cross-border banking groups; consequently, we included each member of foreign or cross-border banking groups separately in the database. The edges between the vertices were derived from the transactions between them (and not from existing stocks). For 1–2 day transactions and the entire network, we created the matrices to describe the graphs by aggregating transactions at intervals of 5 business days. Additionally, for the sake of comparability we also created the graphs at a monthly frequency. This was needed because with segments longer than 1–2 days, we only examined monthly graphs. The main reason for that was that the number of transactions is far lower in such segments, and the size of the networks obtained through 5-day aggregation proved to be too small. We examined transactions longer than overnight in three groups: 3 days to 1 month, 1 month to 3 months, and transactions exceeding 3 months. We established each group by taking two important aspects into account. On the one hand, we needed a sufficient number of vertices for our analysis, while

we also needed to consider, to some extent, the different roles of the transactions with different maturities. We added up the signed forint values of the spot legs of the swaps among the individual actors, assigning a positive value to a transaction where the bank received currency on the spot leg, and a negative one in the opposite case.

4. Methodology

In this chapter, we briefly explain the network theory tools that we used in our analyses (the methodology is identical to that used in our previous paper (*Banai et al. 2013*)). Our aim is to provide a straightforward definition of the indicators characteristic of the network, occasionally at the expense of mathematical precision. Specific results concerning the FX market will be discussed in the next chapter.

4.1. Adjacency matrices and components

Let us assume *N* banks to be given, and *W* to be an NxN matrix wherein element $W_{i,j}$ indicates the forint amount given by bank *i* to bank *j* on the spot leg. In fact, *W* is a matrix including *bilateral exposures* in forint. Each element of the matrix *W* is non-negative and it is not necessarily symmetric, and we assume every element along its main diagonal to be zero. The matrix thus obtained defines a weighted and directed graph without loops,³ where the weight of each edge determines the size of the claim, and the direction determines the direction of the cash flow. Note that where banks *i* and *j* have mutual claims on each other, we use a netted claim obtained as a signed sum of the claims. In analysing the network, the question is often simply whether two actors are connected or not, the size and direction of their connection is less important. This is because the formation of a connection in itself may provide relevant information, while facilitating the understanding and interpretation of the indicators. Additionally, it is important to consider the fact that FX swaps are secured transactions. Consequently, direction is less significant than with commonly analysed unsecured markets.

Let A be an adjacency matrix representing an undirected and unweighted network, i.e.

$$A_{i,j} = \begin{cases} 1, W_{i,j} + W_{j,i} > 0 \\ 0, \text{ otherwise} \end{cases}$$
(1)

The inequality $W_{i,j} + W_{j,i} > 0$ in the above definition will be satisfied exactly when an edge leads from *i* to *j* or from *j* to *i*, i.e. matrix *A* truly represents an undirected network. For example, with the 1–2 day market, the matrices were created by

³ Loop: an edge starting from and ending in the same vertex.

aggregation at intervals of 5 business days, i.e. by obtaining a signed sum of the daily transactions. For aggregation purposes, we did not consider bank holidays in the USA, Switzerland and Europe to be business days because the volume of trade is significantly lower on such days. In calculating the network indicators discussed below, we generally require the network to be connected in some sense.

4. 2. Network indicators

Network size

One of the most general characteristics of the network is its size, which indicates the number of banks participating in at least one transaction in a given period either as borrowers or lenders.

Degree

In a directed graph, the indegree (outdegree) of a vertex i is understood as the number of edges leading to (from) that vertex.

With undirected networks, the degree of i is the number of vertices connected to it. More precisely, if the degree of vertex i is indicated by f(i), then

$$f(i) = \sum_{j=1}^{N} A_{i,j}$$
⁽²⁾

Degree is commonly included among centrality indicators, i.e. characteristic quantities which are supposed to describe the importance of the role that a given vertex plays in the network. Another key quantity is degree distribution. This function indicates the frequency of a given degree value. In networks occurring in reality, degree distribution often follows power law. More precisely, if the frequency of degree k is indicated by p(k), then

$$p(k) = ck^{-\gamma} \tag{3}$$

where *c* is a normalising constant and γ is a positive number, mostly in the interval of [2,3]. Such graphs are called scale-free networks (*Barabási–Albert 1999*). Networks occurring in reality mostly include a large number of low-degree vertices and few higher-degree vertices.

Average path length, diameter and mass function

The distance between vertices u and v is understood as the sum of the weights of the edges along the shortest path between them. In our analyses, we will always consider the largest connected component where we want to use the shortest distance or some of its functions. In the rest of this paper, let d(u,v) indicate distance. In an unweighted graph, d(u,v) will indicate the minimum number of steps required to be taken to get from u to v. Average path length is defined as the

average of such distances, and the diameter of the network as the maximum of such distances.

We also introduce a measure called mass function, which indicates the proportion, relative to all shortest paths, of the shortest distances that will be less than or equal to a given constant k (k=2, 3, 4, 5). Obviously, the function increases in the k parameter, and where k equals the diameter of the network, the result will be 1, since every shortest path is either less than or equal to the diameter.

Closeness^₄

The closeness of a vertex u is the inverse of the length of the path to vertex v, which is at the greatest distance from it. More precisely, if closeness is indicated by c(u), then

$$c(u) = \frac{1}{\max_{v} d(u, v)} \tag{4}$$

The max_v d(u,v) quantity itself illustrates the number of steps required to get from

u to any vertex. Inversion is necessary because we want the closeness of a vertex to be the higher the more central the vertex is to the network, i.e. the fewer steps are required to get to any vertex from it. It is for this property that closeness is considered to be a centrality indicator as well.

Betweenness

Betweenness indicates the number of shortest paths that include a given vertex. We do not count the shortest paths starting from or ending in the vertex concerned. In order to compare betweenness indicators across networks of different sizes, we need to divide the indicator by the maximum number of shortest paths, which for *N* vertices will give

$$\frac{(N-1)(N-2)}{2} \tag{5}$$

The above formula will determine the maximum number of shortest paths that may include a given vertex without starting or ending in that vertex.

Density

Density indicates the ratio of the number of edges to the number of all possible edges. The number of all possible edges in an undirected network of *N* vertices is

$$\frac{N(N-1)}{2} \tag{6}$$

⁴ We also calculated another version of the average closeness indicator. In that indicator, we inverted the average of the shortest paths rather than their maximum. However, we obtained similar results in both cases.

as every vertex may be connected to a maximum of N-1 vertices, but that would include all edges twice. Note that in the case of a directed network, the above formula is modified to N(N-1).

Clustering coefficient

The clustering coefficient of a given vertex shows the ratio of the number of edges between its neighbours to the possible number of edges between its neighbours. In other words, the average clustering coefficient determines the probability of the neighbours of any vertex being connected with one another as well.

Affinity function

The affinity function indicates the average degree of the vertices connected to vertices of a given degree. That is, the function assigns a number to every degree occurring in the network. If the function increases monotonically, vertices of a higher degree will also be connected to vertices of a higher degree, i.e. the key actors will be directly connected to one another. Otherwise vertices of a higher degree will only be connected to vertices of a lower degree, i.e. larger actors will only deal directly with smaller banks.

The small world property

In networks exhibiting the small world property, the average shortest path between vertices is low relative to the size of the network. Additionally, the average shortest path is proportional to the logarithm of network size, i.e.

Average shortest path =
$$c \cdot log(N)$$

where *c* is constant.

4.3. Random graphs

One of the longest established and most studied random network models is the socalled Erdős–Rényi graph (*Erdős–Rényi 1959*). The scheme involves a fixed *N* number of vertices, where two different vertices will be connected with a probability of *p* and will not be connected through an edge with a probability of *1-p*. Primarily, we would like to use the Erdős–Rényi network to compare its average clustering coefficient with that of the graph obtained by us. If the two indicators are close, i.e. their ratio ≈1, we may conclude that our network has no meaningful structure. The question arises how we should construct an appropriate Erdős–Rényi model for a given graph. Obviously, the number of vertices must be identical, and the value of *p* should be selected as follows:

$$\rho = \frac{average \ degree}{N-1} \tag{7}$$

Since our database only includes transactions reported by Hungarian actors, we will slightly adjust the above model because in our network the probability of an edge between two foreign counterparties is zero. Let us consider the subgraph that only includes domestic actors and the edges between them. Let $G_{_{DD}}$ indicate the Erdős–Rényi graph belonging to this network. Additionally, let us create another Erdős–Rényi model for a bipartite graph representing domestic–foreign relationships. A bipartite graph includes two disjoint sets of vertices (domestic and foreign actors), where the edges may only connect domestic and foreign vertices, but neither two domestic nor two foreign vertices. The random graph scheme is modified in that we will only try to connect *i* and *j* vertices with a probability of *p* where *i* is a domestic actor and *j* is a foreign actor. Finally, let us unite the network so obtained and graph $G_{_{DD}}$. In the rest of this paper, we will refer to the graph created this way as a modified Erdős–Rényi network.

5. Results

In the analysis of the overnight market, several findings raised the possibility that the dynamics observed may be caused by the increased prominence of using longer markets (*Banai et al. 2013*). Further analyses suggested that probably this was not



the case, but in the following we explore the specific features of the overall market in detail. In the previous chapter, we described the mathematical tools that we used to map the network structure of the currency swap market. The set of indicators enable us to gain an understanding of the key properties of our network and their development over time. In our calculations, we also examined each indicator separately for segments of various maturities. Additionally, our results include the development of network statistics that ignore maturities and thus describe the overall market.

Our analysis of short markets showed that network size, i.e. the number of banks trading in the given period, is far from being constant (Figure 3). For a while, size was perceivably growing as foreign currency lending took off. From summer 2007 onwards, a series of crisis events could be read from size dynamics. The bankruptcy of Northern Rock caused a break, the so-called decoupling period brought growth, and then the Lehman failure led to a fall. From the second half of 2010, another downward trend emerged, which continued to the end of the period observed. In the networks of segments of longer maturities, the number of vertices fell significantly short of the 1–2 day segment, despite the longer frequency. At the beginning of the period, size exceeded 100 in the 1-2 day network and was still at 70 at the end of the period at a weekly frequency. At a monthly frequency we obtained even higher values, with a maximum at 140 and a size approximating 100 even at the end of the period. This is consistent with the need for actors to renew shorter transactions significantly more frequently. It is seen that in autumn 2008, the number of vertices suddenly dropped in all three long segments. Late 2009 and early 2010 marked the beginning of a more spectacular downward trend. That is, in periods when the size of the short market contracted, a similar trend emerged in longer markets as well. This disproves the possibility that the increased prominence of transactions with longer maturity may have caused the size of the short market to contract. At the same time, it supports the position that many actors were forced to exit the foreign currency/forint FX swap market by an increase in risks, despite credit risk remaining low. It is worth noting that the transactions entered into with the MNB in accordance with the forint conversion announced in November 2014 did not result in any significant changes in the number of actors in November. Only the 1 to 3-month market showed some increase. This may be due to the fact that counterparties will receive currency from the MNB, which they can use to close their market swaps among other things, with a delay, between 2015 and 2017. Consequently, the size of the market is expected to contract only later.

In terms of our research, the first and most striking network property of the FX swap market is that the graph obtained is often not connected. When we consider the network formed on a single trade day, the network is found to be disintegrating into several (in some cases more than 10) separate parts. For our analysis, it is

important to obtain a connected network, since the calculation of certain centrality indicators only makes sense if we do. It was an important question in the case of the 1-2 day market as well to find the sufficiently short frequency where a connected network is formed. Due to the small size, this problem becomes even more significant with longer segments. We decided to use a monthly frequency in such cases. A lower frequency would already "conceal" many events, and as such would not be a suitable choice. However, it is apparent that, especially with transactions exceeding 3 months, in extreme cases the largest connected component is comprised of only 20 to 30 percent of the actors. That is why a higher frequency is not recommended either (*Figure 4*).



A key question is what causes this disintegration. In the literature, we have found several references to examples of markets which were not completely connected as networks (e.g. *Berlinger et al. 2011; Bech–Atalay 2008*). Such a level of disintegration may be explained by the fact that a major part of the actors are foreign banks. In many cases, foreign actors mostly trade with their own local subsidiary banks. When the relationship is mutually exclusive, the two vertices will break away from the rest of the network. A further obvious explanation may be the fact that we are unable

to see a part of the network (transactions between foreign actors). Naturally, at progressively higher frequencies, the decrease in the number of transactions in itself increases the probability of separate bank pairs, triangles, etc. being formed. It is important to note that from the beginning of the crisis and particularly from autumn 2008 onwards, the absence of connectivity became increasingly prominent. With the exception of the 1–2 days market, the ratio of the largest connected components to the entire network gradually diminished in every segment. This points to increasing mutual distrust, which motivated participants to enter into longer transactions only with a small group of counterparties. Transactions of longer maturities also involve increased risk, which is why it is reasonable to select counterparties even more rigorously in longer markets.

Although it is possible to calculate some of the indicators even with the network disintegrating, the calculation of most centrality indicators requires at least weak connectivity. For that reason, we will consider the largest connected component throughout the rest of our investigation. As seen above, in some exceptional cases, the greater part of the network falls outside of the greatest connected component. Consequently, with the longest markets the conclusions that can be drawn from the network indicators are limited. In the rest of this paper, such conclusions will primarily be used to support the conclusions drawn in the short market.



We examined the extent to which the graph can be considered random. For this purpose, we compared the graph of the swap market to random graphs generated using two methods. On the one hand, we compared it to a random Erdős–Rényi graph, the average degree of which was identical to that of the swap market graph. On the other hand, we also considered a modified random Erdős–Rényi graph, where we assigned varying probabilities to the edges between domestic–domestic, domestic–foreign and foreign–foreign vertex pairs. In the modified random graph, a connection is established between two foreign vertices with a probability of 0.

We calculated the average clustering coefficient for all four graphs. We found that the level of clustering in the swap market graph significantly exceeded the level of clustering in both random graphs for all maturity segments (overall market: *Figure 5*, short market: 3 times larger clustering on average; 3 days to 1 month: 3 times larger clustering; 1 month to 3 months: 2 times; above 3 months: 2 times). Consequently, the swap market network cannot be considered to be random (although the clustering coefficient approximates that of the random graph as maturity increases), and as such it is worth examining in more detail.





Banai et al. (2013) have demonstrated that the network of the 1–2 day market may be referred to as a scale-free network. Accordingly, the degree of individual vertices approximately follows power law. If k denotes individual degrees, then the power function $53^{*}k^{-2}$ will give a fairly good approximation of degree frequency. As a result of the distribution following a power function, the graph includes many low-degree and few high-degree vertices. The small world property may also be captured through the development of specific network indicators. One of them is the ratio of average shortest path length to network size. A low ratio will indicate the small world property. The small world property is also indicated by a clustering coefficient exceeding that of the random graph, or an average shortest path length proportional to the logarithm of network size (Pető–Békési 2009; Newman, 2003). Clearly, the first indicator shows significant differences with segments of various maturities (Figure 6). In the case of the 1–2 day market, the average shortest path length amounts to a mere 3% of the network size in general. However, the indicator significantly increases with longer maturities (3 days to 1 month: 4.5% on average; 1 month to 3 months: 7.5%, above 3 months: 8%), showing an approximation to so-called lattices (Pető–Békési 2009). This is confirmed by the ratio of the average shortest path length to the logarithm of network size. The longer the segment being considered, the less constant the indicator became. Overall, it appears that the





longer the maturities, the less the network can be considered to satisfy the small world property. In networks, the small world property poses a stability risk. Namely, such networks respond drastically to shocks affecting the largest actors, although shocks to small actors have no significant effect (*Albert et al. 1999; Newman 2003*). In times of crisis, central actors tend to amplify and accelerate contagion (*Markose et al. 2010*). Conversely, with random graphs shocks to the largest actors have a smaller effect. The longer the markets, the smaller the chance of contagion. This also follows from the gradually decreasing ratio of elements that are part of the largest component. An analysis of the entire swap market as a single network shows that the two indicators behave similarly to the 1–2 day segment, i.e. the small world property is also satisfied in this respect. This is explained by the fact that within the overall market, 76% of all transactions were made in the overnight segment.

Throughout the period under review, the average degree of the network varied significantly across segments in terms of both trends and levels (Figure 7). In the shortest market at a weekly frequency, a change in the indicator could be observed at both turning points of the financial crisis, i.e. in summer 2007 and autumn 2008 as well. In mid-2007, for a brief period there was a significant decrease in the average degree of the network, and then the indicator moved at previous levels right up to the Lehman bankruptcy. In turn, after the Lehman bankruptcy, the indicator remained below previous levels for an extended period, which was followed by a continuous increase from autumn 2010 (Banai et al. 2013). At a frequency of one month, the change in the 1-2 day segment was also striking at the time of the Lehman bankruptcy and from the second half of 2010. However, the picture is different with the rest of the segments. On the one hand, as we expected, the level of average degree is significantly lower due to differences in size. On the other hand, we also found differences in trends. While the segment of 3 days to 1 month behaved similarly to the shortest market, the graphs derived from transactions exceeding 1 month moved in the opposite direction. With the former, the average degree reached its peak by the end of the period, which means that the weight of relatively important banks increased across the network. With the latter, however, the average degree decreased. This may also indicate banks' increased tendency in the aftermath of the crisis to select the counterparties with which to enter into longer transactions.

Not surprisingly, average shortest path length moved in the opposite direction to average degree. This was also observed for the 1–2 day market (*Banai et al. 2013*), but also applied to the graph including all of the transactions regardless of maturity. Due to the fact that individual actors enter into transactions with an increasing number of counterparties, new edges allow the formation of additional and shorter paths between two vertices. Obviously, the opposite is also true. Fewer counterparties mean fewer variations of possible paths, causing the average shortest path to increase (*Figure 8*). With segments longer than 2 days, average shortest path



length was around 3 in the period under review. The obvious trends and turningpoints seen in the overall market were increasingly less prominent there. It was apparent particularly in the longest segment that although the indicator was highly volatile, no trends could be established in its movements.

An important question relating to the development of degrees is the degree which the neighbours of vertices with various degrees have themselves (Figure 9). In financial networks, banks with a high degree typically deal with counterparties that have a low degree (the phenomenon has been described in *lori et al., 2008; lazzetta et al. 2009*). This was so in the case of the Hungarian 1–2 day FX swap market (Banai et al. 2013), and is also observed in the graph derived from the overall market (Figure 9). One reason for this is the high number of small actors in the network, which drives the most active actor to also become connected to actors with few counterparties. On the other hand, account should be taken of the fact that the actors considered small in terms of the domestic currency swap market include many banks that are prominent internationally as well. This disassortativity is also characteristic of longer markets (Newman 2003), but to a smaller extent. The downward sloping affinity function is still clearly visible in the segment shorter than 1 month; however, in longer segments counterparties have much more homogeneous average degrees. This may indicate that a central role is played by a few actors to a smaller extent.



Note: Axis x shows the degrees of individual vertices, whereas axis y shows the average degrees of neighbouring vertices. For example: in the overall market the vertices neighbouring vertices with a degree of 1 had an average degree of around 37. Source: Own calculations based on MNB data.

The size of network diameter may be dominant in the spread of shocks. As a rule of thumb, in social networks the small world property means a maximum diameter of 6 (*Newman 2003*). In the 1–2 day FX swap market, diameter was found to be rather stable in time. Throughout the period under review, average diameter was 5.2 at a weekly frequency and 4.4 at a monthly frequency. Moreover, at a monthly frequency the lowest value was 4 and the highest was 5, i.e. the indicator hardly changed over time. In longer segments, diameters averaged around 6 and increased as maturities became longer (3 days to 1 month: 5.9 on average, 1 month to 3 months: 6.1, above 3 months: 6.1). Apart from this, longer maturities also involved higher volatility. In such turbulent times as autumn 2008, the diameter of certain graphs exceeded 10 (*Figure 10*). In other words, fewer and fewer connections were formed between banks. While this obviously reduces the risk of contagion, it also means that certain banks would not be able to enter into transactions with a sufficient number of counterparties. As a result, they may have been forced to rely excessively on a single counterparty in longer markets.

In the case of the unsecured forint interbank market, the indicator of average closeness was the quickest to respond to the crisis. Although from a rather high level





of 0.5, it started to decline as early as in 2006, and already moved around 0.4 in early 2009 (Berlinger et al. 2011). All of the networks in our analysis deviated from this in terms of both trends and levels. In the graph of the overall market, the network indicator perceivably changes as problems develop. We could see an increase from late 2006 to spring 2008, followed by a decrease, which was intensified by the Lehman bankruptcy. Therefore, in this case the trend changed earlier than in the 1–2 day market. The sharp decline around the Lehman bankruptcy coincided with that seen in the 1–2 day market, just as the continuous increase from 2010. The two graphs differ primarily in terms of levels. In the network of the overall market, the highest average exceeds 0.4, which is significantly higher than the maximum of around 0.3 measured for the 1–2 day segment (Figure 11).

In longer segments, a decrease in average closeness was also felt in autumn 2008 (3 days to 1 month: 0.29 on average, 1 month to 3 months: 0.2, above 3 months: 0.2). On aggregate, however, the time series appeared stable without any significant changes. This suggests that the periphery of the network exited the market mostly in shorter segments (*Banai et al. 2013*). In the rest of the segments, it is not clear how important exiting actors had been for the network concerned.

The clustering coefficient is a key indicator in terms of stability both at systemic and individual levels. Its movements aptly describe the extent to which cliques are formed in the given market, and how typical it is for the partners of specific banks to enter into transactions with one another. In the period under review, major crisis events led to significant drops in the indicator. For most of the period, the indicator moved between 0.1 and 0.2 in the 1–2 day market, hitting a low of 0.05 at the time of the Lehman bankruptcy (*Banai et al. 2013*).

Not surprisingly, with longer segments the level of clustering decreased (*Figure 12*). This is also indicative of institutions being more rigorous in the selection of their counterparties as maturities become longer. Transactions below 1 month still showed the tendency seen in the 1–2 day segment that clustering increased from mid-2010; however, longer segments behaved differently. Apparently, above 1 month and in particular with maturities exceeding 3 months, values around 0 are not uncommon. In other words, triangles or cliques are not formed in the graph. The clustering coefficient also indicates that the longer the maturities, the less the graphs are characterised by the small world property. The average of the average clustering coefficient in longer segments was as follows: 0.11 for 3 days to 1 month, 0.09 for 1 month to 3 months, and 0.07 above 3 months. In this regard, longer markets are more similar to the random graph. In autumn 2008, a decline in clustering was observed in all maturity segments.

Changes in network structure are also well characterised by the dynamics of the density function. In the 1-2 day segment, at a weekly frequency the indicator remained relatively stable right until summer 2010. As with the other indicators



examined so far, the second half of 2010 also marked a turning point here. The significant increase in density may be attributed primarily to the loss of vertices with low degrees. This is because in such cases, while there is hardly any change in the number of connections formed (the numerator), the number of possible connections (the denominator) decreases significantly. The same result is obtained from an analysis of the 1–2 day market at a monthly frequency (Figure 13). The indicator is relatively low at around 4%, which is not uncommon with financial networks. The densities of the networks with various maturities hit their respective lows at the time of the Lehman bankruptcy, which means that the smallest number of connections relative to the possibilities were formed at that time. Subsequently, however, longer segments show significant differences versus the 1–2 day market in terms of both levels and appearance. Most strikingly, the level of density increases as maturities get longer. This is attributable to the fact that as a result of a decrease in size, the number of potential connections, i.e. the nominator will be significantly smaller. Although transactions within 1 month still show some stability, the indicator moves on an upward trend from 2010 in this segment as well. Moreover, the indicator is much more volatile than with the shortest transactions. A gradual increase was also observed for the two longest segments towards the end of the period under review, but the level reached by the end of the period is not outstandingly high in a historical comparison. In other words, the ratio of the connections formed was not



particularly high even with the contraction in network size, despite the fact that the number of possible connections increases at a rate of the size squared. This may be another indication that trust weakened in longer segments as well.

6. Conclusion

During the economic crisis that started in 2007, serious disruptions occurred several times in the operation of various financial markets. Certain markets dried up completely, and central bank intervention was needed to ensure that the consequences of their loss caused a minimum amount of damage for the banking system. One particular feature of the crisis was that serious disruptions occurred worldwide even in the secured FX swap market. For that reason, central banks entered into a series of bilateral agreements with one another, temporarily assuming the role of the market. As *Banai et al. (2013)* demonstrated in the context of the short-term FX swap market, in Hungary the disruptions were clearly shown by the network structure of the market in addition to commonly used market indicators such as implied yield, liquidity indices, and turnover. The particular structural characteristics of the graph of the short-term market occasionally exhibited strong volatility during the crisis. In their analysis of the largest connected component, the authors found that the properties generally satisfied for financial markets were also

characteristic of this market. Like other markets, the one-day FX swap market exhibits the small world property, i.e. it is possible to reach any vertex from any vertex in a relatively small number of steps. We have seen that the degree distribution of the network follows power law. Most participants have a relative small number of connections, and there are only few large actors with outstandingly high degrees.

In this study, we aimed to develop an understanding of the properties of networks derived from FX swaps of longer maturities (see the Appendix for a comparative table), and to obtain a picture of the overall market. This also provided us with a more complete picture of the short-term market. Regarding the short market, one of our findings was that the number of participants decreased significantly, with a particular decline in the activity of marginal actors. Through an analysis of the overall market, we confirmed that this was not attributable to longer transaction maturities, since graph sizes decreased following the start of the crisis in longer markets as well. With the longest markets, however, the type of exiting actors is not certain.

In the one-day market, we have seen that the network was not connected at a daily frequency, and that there were isolated parts even at a weekly frequency. This property was intensified with longer transactions despite the monthly frequency used. At a monthly frequency, often only 60–70% of the vertices formed a connected component. With the longest transactions exceeding 3 months, occasionally only 30% of the actors were connected. During the crisis, this property became particularly pronounced in the case of the longest transactions, which indicates that certain banks had the confidence to enter into long transactions only with a very small group of institutions.

In the case of the short-term market, we have seen the network to exhibit the small world property that is characteristic of financial networks. This also means that the market is especially sensitive to the behaviour of a few actors, which presents a stability risk *(Albert et al. 1999; Newman 2003)*. The analysis of longer networks showed this small world property to be less prominent. It definitely applied to networks derived from transactions between 3 days and 1 month; however, the graph of transactions above 3 months increasingly approximated a random graph as the crisis developed. As the number of actors decreased, the network became less and less clustered, and groups gradually disappeared. This may indicate increasing distrust among the actors.

In our study, we paid particular attention to the network derived from the overall market regardless of maturities. Although a distinction between maturities was required due to differences in functions, we were also curious to see the behaviour of the graph of the overall market. As expected, the trends observed here followed those of the one-day market, since a vast majority of the transaction volume is associated with that market. This confirmed our assumption that a segmentation of the overall market was reasonable. This enabled us to identify different trends for transactions of different maturities.

	1-2 days	3 days-1 month	1 month-3 months	>3 months
Size (monthly graph)	 minimum 94 116 on average "Decoupling" visible decreases from Autumn 2008 	 minimum 50 73 on average "Decoupling" visible decreases from Autumn 2008 	 minimum 22 44 on average "Decoupling" not visible decreases from Autumn 2008 	 minimum 18 41 on average "Decoupling" not visible decreases from Autumn 2008
Proportion of largest connected component	• usually >90%	• usually >90%	usually >60%once decreases to 40%	 usually >60% twice falls under 30%
Type of graph	small world property • degree distribution follows power law • shortest path/size small • shortest path/log(size) constant • downward sloping affinity (disassortative) • diameter (5 on average) • mass function, high ratio of short paths • average clustering high	small world property • shortest path/size small • shortest path/log(size) constant • downward sloping affinity (disassortative) • diameter (6 on average) • mass function, high ratio of short paths • average clustering high, but lower than of 1-2 days	small world property less satisfied • according to shortest path/size it is closer to lattice • according to average clustering it is closer to random graph	no small world property
Effect of crisis	 degree dropped in Summer 2007 and Autumn 2008 distances grew (shortest path, diameter, mass function, average closeness) clustering fell size decreased in Autumn 2008 direction of cash flow between residents and non-residents changed in 2009 	 the graph is smaller and denser from 2008 distances increased in Autumn 2008 (shortest path, diameter, mass function, average closeness) 	• diameter increased in Summer and Autumn 2007	 distances increased in Autumn 2008 average degree and mass function increases since 2007 already direction of cash flow between residents and non-residents changed in 2010
Exit of the actors on the periphery	visible • size • degree • shortest path • mass function • average closeness • average clustering • vertices of clustering of 0 and 1 • density	visible • size • degree • shortest path • mass function • average closeness • average clustering • vertices of clustering • density	• happens earlier (from end-2008 to end-2010)	not visible

Appendix: Comparative table of the graphs for specific segments

References

- Albert, R. Jeong, H. Barabási, A. L. (1999): Error and attack tolerance of complex networks. Nature, Vol. 406, pp. 378-382.
- Balogh, Cs. Gábriel, P. (2003): Bankközi pénzpiacok fejlődésének trendjei. Magyar Nemzeti Bank Műhelytanulmányok 28. szám, 2003. november.
- Banai, Á. Király, J. Nagy, M. (2010): Az aranykor vége Magyarországon, Külföldi szakmai és lokális tulajdonú bankok – válság előtt és válság után. Közgazdasági Szemle, 57. évf. 2. sz.
- Banai, Á. Kollarik, A. Szabó-Solticzky, A. (2013): Az egynapos FX-swap piac topológiája. Magyar Nemzeti Bank Tanulmányok 108., 2013. november
- Barabási, A. L. Albert, R. (1999): Emergence of Scaling in Random Networks. Science, Vol. 286.
- Bech, M. L. Atalay, E. (2008): The Topology of the Federal Funds Market, Federal Reserve Bank of New York Staff Reports, 354. szám 2008. november.
- Bergsten (2008) http://blogs.ft.com/economistsforum/2008/07/trade-has-saved-americafromrecession/
- Berlinger, E. Michaletzky, M. Szenes, M. (2011): A fedezetlen bankközi forintpiac hálózati dinamikájának vizsgálata a likviditási válság előtt és után. Közgazdasági Szemle, 58. évf. 3. sz.
- BIS (1998): *Report on OTC Derivatives: settlement procedures and counterparty risk management*. CPSS Publications 27. szám, 1998. szeptember.
- Cocco, J. F. Gomes, F. J. Martins, N. C. (2003): *Lending relationships in the interbank market*. (http://ssrn.com/abstract=568704i.)
- Csávás Csaba Kóczán Gergely Varga Lóránt (2006): *A főbb hazai pénzügyi piacok meghatározó szereplői és jellemző kereskedési stratégiái*, Magyar Nemzeti Bank Tanulmányok, 54. szám.
- Csávás, Cs. Szabó, R. (2010): A forint/deviza FX-swap szpredek mozgatórugói a Lehman-csőd utáni időszakban, Hitelintézeti Szemle 2010. 6. szám.
- Erdős, P. Rényi, A. (1959): On Random Graphs. I, Publicationes Mathematicae 6: 290–297.
- Fábián, G. Mátrai, R. (2012): A nemkonvencionális jegybanki eszközök magyarországi alkalmazása, MNB-Szemle 2012. június.
- Iazzetta, I. Manna, M. (2009): The topology of the interbank market: developments in Italy since 1990, Banca d'Italia Working Papers 711. szám, 2009. május.

- Iori, G. De Masib, G. Precupc, O. V. Gabbid, G. Cadarelli, G. (2008): A network analysis of the Italian overnight money market, Journal of Economic Dynamics & Control 32. szám, 259-278. old.
- Lublóy, Á. (2006): Topology of the Hungarian large-value transfer system. Magyar Nemzeti Bank Tanulmányok, 57. szám.
- Markose, S. Giansante, S. Gatkowsk, M. Shaghaghi, A. R.(2010): Too Interconnected To Fail: Financial Contagion and Systemic Risk In Network Model of CDS and Other Credit Enhancement Obligations of US Banks. COMISEF Working Paper 033. szám.
- Newman, M. E. J. (2003): *The Structure and Function of Complex Networks*. SIAM Review 45. szám, 167-256.
- Páles, J. Kuti, Zs. Csávás, Cs. (2010): A devizaswapok szerepe a hazai bankrendszerben és a swappiac válság alatti működésének vizsgálata, Magyar Nemzeti Bank Tanulmányok, 90. szám.
- Páles, J. Varga, L. (2008): A magyar pénzügyi piacok likviditásának alakulása mit mutat az MNB új aggregált piaci likviditási indexe?, MNB-Szemle 2008. április.
- Pető, R. Békési, L. (2009): Az Európai Unió grafológiája: az Európai külkereskedelem elemzése a gráfelmélet segítségével, Tudományos Diákköri Konferencia dolgozat.
- Soramäki, K. Bech, M. L. Arnold, J. Glass, R. J. Beyeler, W. E. (2006): *The Topology of Interbank Payment Flows*, Federal Reserve Bank of New York Staff Reports, 243. szám.
- Watts, D. J. Strogatz, H. S. (1998). *Collective dynamics of. 'small-world' networks*. Nature, Vol. 393, No. 6684.